Local Compatibility Constrained Financial Market Dynamics with Asymmetric Agent Interactions *

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Abstract

We study the effects on single good market price time series induced by various types of agents’ social interactions and information propagation between them. We model them by different topologies of agents network and diverse levels of asymmetry in the information transfer. Agents can buy or sell one unit of a good (small investors) or more units of a good (big investors) or do not take any action at all in the single time step. We generalize existing models by not taking all the interactions between the agents to be strictly positive, but just constraining them to be mainly positive: agents are mainly following their neighbors, but not necessary in all of the cases (in some situations they can form the opinion that the prices are on the exploding path and decide not to follow the decisions of the other agents he is in contact with). By the means of the previous concepts, we extend the model used in [1] and other similar models in the literature. We emphasize that the model possesses the information asymmetry in the sense that some agents are influenced more by the others and interactions are not symmetric (for example, bigger investors will have more influence on small ones than vice versa). In this framework, using the standard microstructure bid and ask rules, we generate as output price time-series and calculate their volatilities and corresponding correlation functions. We study them using the different instruments from statistics and statistical physics and we compare them with the real market price time series properties.

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1 Introduction

Understanding the dynamics, i.e. the reasons for econometrically detected properties of financial markets price and volatility time series (that are just their kinematical characterisation and represent simply a collection of the time series properties), is one of the most important arguments in the financial markets microstructure theory and numerical analysis (usually Monte Carlo simulations) in the current research. Microstructure studies are also crucial for many practical purposes in different areas: for example, they have direct implementation and enormous consequences on financial risk management (ultra-high frequency correlations in data versus market transaction costs, for example), regulation of financial institutions and quantitative option pricing and hedging in general (see, for example, [8] and [9]). Better understanding of the prices statistics, especially large deviations, and mechanisms of their formation (and, consequently, their volatility time series formation) in the "market game" with many interacting players would lead to more efficient markets with the smaller transaction costs.

Different measures of financial risk exist; all of them depend on statistical properties of asset prices (see, for example, [11], [10], [12] and [13]). Standard models (like Black and Scholes model, see [28], [29], [11]; for more correct, non-Gaussian, option pricing see [10], for example, or any advanced textbook on option pricing beyond the standard log-gaussian Black and Sholes model) are underestimating the possible price fluctuations and eliminating the volatility "bursts" from the price evolution.

The most striking possibility that understanding of the mechanisms underlying the market dynamics could give us is to develop methods for continuous monitoring of eventual precursors of large price deviations and volatility clustering and eventually intervene to prevent large price deviations from the fundamental values (especially important for central banks intervention policy in the foreign exchange markets) and negative effects from "nervous markets".

Risk estimators created in accordance with the better known market price formation mechanisms could be than used to set up in a more correct way the levels of the wished risk by single "careful" traders and hedge funds. If applied by many traders, this kind of procedures could make markets more rational and less volatile, especially by the elimination of at least some of the largest price oscillations and speculation possibilities. It is striking that still the most of traders, even in the biggest banks situated in the City of London, are still using the so called "technical analysis" (see the study done by Taylor
at all. [?]). More "fundamentalists" (rational traders), especially for the foreign exchange markets, would lead to the elimination of the large price deviations from the range of their possible fundamental levels and smoother economy in general, so it could have interesting positive macroeconomical aspects.

Of course, no one can expect to have a complete predictability of the price evolution, but some relatively big events could be eventually "detected" in advance and prevented in some cases (see [14] and also [18] for other anomalies). Even years before, there can be individuated precursors like consistent raising of the price fluctuations (in the series of the daily fluctuations) and strong correlations in fluctuations of different market indices (the last is especially manifested during the panic events very near to the market crash date).

The role of market makers and big banks in general that, by the nature of they job and size of financial operations, posses the large quantities of private information and are willing to exploit them invoke the concept of the large information assymetry between different financial markets players and players on different financial scales. Classical is the example of, say, British Petroleum, that is asking from a bank to buy on the forward market few billions of dollars. This kind of bank exposure on the information (maybe with many clients like Shell or BP) is giving them a sicure way of earning by speculation using the private information in the forward exchange and other market branches, for example. So, big players are influencing more other players that are not in the position to get the private information. This kind of operations executed by big multinational companies are impossible to be controlled by any kind of central bank interventions and are as well disconnected (are on the much smaller time scales) from the fundamental macroeconomical variables movements. They, and other agents reactions on them, could induce different price and volatility dynamics and price equailibrium formation from the standard log-gaussian one. We see that the main roll is played by the asymmetry in the market players dimensions and influences on the other market participants. In this paper we will try to explore those facts in order to see if they are inducing the real prices dynamics. Of course, today, in a world of large international capital and highly substitutable assets, policy makers interventions are not as important as in seventies, for example, but are still influencing trough the signaling channel mainly, see [?]. This strong intension to smooth the economy (especialy, forex rates time series) could be a very intereseting subject for the future studies. Unfortunately, many data on the central banks interventions are still secret.

In principle, market dynamics studies could improve the knowledge about the probabilities for given large price drops or raises in given future time
intervals \(^1\). The final goal would be to understand the stochastic dynamical processes that govern this kind of systems and, consequently, to forecast more precisely the intervals in time and in event intensity we have mentioned. On one side, there are already very sophisticated tools developed (multifractal analysis in statistical physics) for the analysis of time series and extraction of the most of the properties that are allowing us to determine the price changes probabilities and volatility clustering intervals and uncover the underlying dynamical mechanisms of price time series creation; also, there are dynamical systems that are producing as their outputs signals very similar to price time series, on the other side. Here, the idea is to use the known more or less simple models from statistical physics\(^2\) and their good properties and to modify them as slightly as possible to produce the typical market behaviour.

If we want to know with highest precision the level of agreement between a market model output price time series and the real market cases, we need to use the sophisticated techniques developed for the studies of multifractal time series, in order to extract as much as possible information from a theoretical model output as well as from the real markets, and than to compare them. The matching of model characteristics with the real markets ones could be in principal obtained by the model modifications or a single model parameters modifications.

Even if crashes could be regarded as a natural corrections that are bringing the market to a “realistic” state, some sets of events are already known to be precursors of a significant price drops and central banks and governments are intensively and continuously working on economy smoothing! It is important to stress that large price fluctuations are not always connected to the emerging of bad/good news and their propagation, or to significant variations of fundamental economic variables – their are rather a consequence

\(^1\)Let us present this idea following the next example, that everyone is probably familiar with. Financial markets analysis (in the absence of eventual relevant information) is essentially similar with the earthquakes prediction case. We can know that certain regions of the Earth will be subject of big events with some probability in the following period of time, but we can not say exactly when and exactly of which intensity. This similarity with a financial markets is not a case, the same mathematical (fractal) properties are characterizing and similar dynamical processes are governing the two systems.

\(^2\)Here is the right place to emphasize, in a very rough but essentially correct way, one important result in statistical physics that says that the properties of a many body systems do not depend significantly from the details of the dynamical system structure. The model behavior is governed by some global model characteristics like the locality of the subsystems interactions and the model topology. We can make no significant improvements by complicating the system modeling enormously. Anyhow, it is never possible to simulate a real system in all of its details. What is important is to see what are the model ingredients (and their combinations) that are leading to the wished model properties.
of collective emerging phenomena like crowed effects or herd behavior in the agent networks: in the framework of the interacting agents models financial markets can be seen as a complex interactive systems that posses a non-symmetric communication network, see [17], [26], [25], [27] and [24]. Every single agent can buy or sell one (discrete models) or more units of a given good (in the limit of large number of units we have continuous models).

The paper is divided into other four sections. In section 2 we review briefly the properties of the prices in the real markets. In section 3 we present some basic models of interacting agents in the financial market. In section 4 we go deeper in the local compatibility constrained model and we present the numerical simulations results. Section 5 concludes the paper.

2 Price Time Series Properties

The distributions of market returns, i.e. logarithmic price differences

$$\ln\left(\frac{P(t + \Delta t)}{P(t)}\right),$$

on the time scales $\Delta t$ that range from minutes up to days (see [8], [15], [16], [19], [20] and [12]) are leptokurtic with fat tails that are respecting the powerlow

$$P(\text{return} > x) = Cx^{-\mu},$$

where $\mu$ is in the range $2 - 4$.

So, Brownian motion (the standard, uncorrelated one $^3$) is unable to explain even qualitatively important features of price dynamics. Very accurate analysis (see [19]) of the so called $\Delta$-trading time $^4$, i.e. in the economical terms are considered speculators (patient investors) that modify their portfolio only when a fluctuation of a size $\Delta$ appears in the price sequence, in the high frequency data from the most liquid, practically continuously traded currency exchange market, the US dollar/Deutschemark market, that discard the standard “random walk” hypothesis clearly; the authors show that the order one Markov process mimics properly this currency exchange market.

$^3$Standard, uncorrelated Brownian motion is the Hurst exponent $H = \frac{1}{2}$ “random walk”, see [28], [29], [20], [10] and [30]. This time series correspond to the one used in the Black and Scholes model.

$^4$This method have an origin in the so called Kolmogorov $\varepsilon$ entropy in statistical physics, and is one of the most powerful methods for the fractal (time) series analysis.
The distribution of returns is strongly non Gaussian, its shape continuously depends on the return period $\Delta t$, and slowly converge to Gaussian distributions as $\Delta t$ increases. For $\Delta t$ large enough (around few months) quasi-Gaussian return distributions are observed while for small $\Delta t$ the return distributions have a strong kurtosis.

Volatility clustering appears in price time series: periods of large volatility tend to be followed by periods of large volatility $^5$.

Another important characteristic of the price and their volatility time series is the so called leverage effect, first observed by Black (see [21]): past price changes and future volatilities are negatively correlated, i.e. financial markets are more active after a price drop and tend to calm down after a price raise. This is the effect that is always present, but is highly visible in the case of option markets and stock indices. Leverage effect mirrors in the anomalous negative skew in the distribution of price changes (returns) as a function of time. In the case of option prices the effect is usually the strongest. For individual stocks the effect is moderate, but decays over $\approx 50$ days, while in the case of stock indices leverage effect is much stronger, but decays $\approx 5$ times faster.

Statistical anomalies that we listed are common for all the financial assets and all the financial markets, i.e. independently if the asset is an individual stock or stock index, option price,...

3 Financial Market Interacting Agents Models

Agents models of financial markets can be seen as a complex interactive systems that posses a (not) symmetric communication network, see [17], [26], [25], [27] and [24], where traders are buying and selling $m$ units of a financial asset.

$^5$The daily temporal correlation function of the volatility can be fitted by an inverse power of the leg, with a small exponent $(0.1 - 0.3)$, see [8].
3.1 Short History of Mathematical modeling of the market

The simplest model consists of a set of N traders, forming a quadratic lattice on a torus, each of which have a possibility to buy or sell one unit of a good, while the network topology is very simple: he interacts (exchange information) only with the nearest neighbors, and all the interactions are positive and have the same value. As the interactions are localized maximally (just the nearest neighbors are exchanging information), we expect smaller correlation between the states (buy/sell) of far away agents. In this way a single agent is influenced positively by the agents he “sees”, i.e. he tends to optimize his gain by following his neighbors’ decisions. Mathematical modelization of this simple model is given by the following payoff function 6:

\[ V = V(s_i, J_{ij}) = \sum_i J_{ij} s_i s_j, \]  

(3)

where the sum is taken with respect to the nearest neighbors (this is the single-agent hamiltonian that he tend to maximize). What makes the model are the characteristics of the \( s_i \) and \( J_{ij} \), i.e. the values the state \( s \) of the agent \( i \) can take (+1 and −1) and \( J_{ij} \) matrix that is telling us in witch relation are agent \( i \) and agent \( j \). This matrix contain all the essential information we incorporated in the model: the interaction intensity that is defining the topology of the agent network and the symmetry or asymmetry of information flows between the agents (traders). In this simple model, topology (subnetworks defined by the connected, i.e. interacting agents) is the simplest possible; as agents are on the square lattice on the torus, they can ”see” just their four nearest neighbors and react on their actions. All the agents are the same in principle, there are no privileged agents: for example, those that could influence more on the other agents, or that could move the market more by buying or selling more units of traded good (like market makers do, for instance, as we commented in the introduction), or having the both of mentioned possibilities at the same time, or some other possibility or extra information about the future price. All the model characteristics we just cited translate into a symmetric interaction matrix \( J_{ij} \), where all the

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6In statistical mechanics and dynamic optimization theory this kind of functions are called Hamiltonians. Hamiltonian we just wrote corresponds to the famous Ising model, created in the beginning of the past century to explain the magnetization phenomena. We see that there can be a complete analogy between the physical systems and financial markets that reflects in the same underlying mathematics.
elements are or zero (correspond to non interacting agents, i.e. those that are not the nearest neighbours) or some positive constant $J$ (as all the agents are influencing the others, here just the nearest neighbours, in the same way and, automatically, we have symmetric information flows). The positivity of $J$ is explainable in economic terms like the local complementarity between the agents. They are trying to imitate each other (see [5], [7], [22] and [1]) in order to maximize the products $s_is_j$ in the hamiltonian, and as the $J$’s are positive, the whole sum.

Agents could have some intrinsic attributes that can show their financial power and influence on the others. The number of connections with the other agents they have and the “force” intensity that they exercise on the other agents to make certain decisions regarding the buying, selling or waiting are some of them. Topology of the single trader communication network (exchanging of information) is defined by the set of the agents on whom this pressure he exercises or from whom he receives information and consequently conform to it by the optimization of proper term in the hamiltonian.

We can generalize the last formula by taking

$$V = V(s_i, J_{ij}) = \sum_i (C_i M s_i + J_{ij} s_i s_j + \varepsilon_i), \quad (4)$$

where

$$M = \frac{1}{N} \sum_i s_i, \quad (5)$$

and $\varepsilon_i$ represent noise (random shocks) in the payoff function (hamiltonian) and simulate an idiosyncratic shocks in the payoff function (hamiltonian) and simulate an idiosyncratic decision of the agent $i$ (that reflects a specific information set available to that agent, see for example [7] and [23]; the concept of noise terms in hamiltonians is very old and spread in theoretical physics).

In the more general case the first term in the sum in the last formula can be taken for some agents with a positive sign ($C_i > 0$) – and in that case we speak about the strategic substitutability because the agent belongs to the majority group – or with a negative sign ($C_i < 0$) – in which case we speak about the strategic complementarity as the agent belong to the minority group (see the detailed discussion in [1]).

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7 This arise as a natural fact in the framework of Ising model in statistical physics.
4 The Local Compatibility Constrained Model: Description of the Model and the Computer Program

The model that we have choose as basis to write the computer program is similar to the model used in [1] and other similar approaches and extends them in two directions with respect to the ideas developed in [7], [5] and [1]. We are using the single-agent surplus function of the type (the written code is very flexible and similar functions can be introduced in the program by the relatively simple modifications):

\[ V_i = V_i(s_i, J_{ij}) = \sum_i (aM s_i + J_{ij} s_i s_j + b \varepsilon_i), \]  

(6)

where \( M = \sum_i^N s_i \), \( \varepsilon_i \) are random shocks, while \( J_{ij} \) satisfy very particular conditions, as we will see now. Here, the maximization takes place with respect to the possible choices of decisions \( s \) for the given agent \( i \). We have two types of agents (traders or investors) – some percent, \( r \), of big investors that can buy or sell \( m > 1 \) or zero units of a traded good and \( 1 - r \) small investors, that can buy or sell just one or zero units of a good. This gives us a possibility to study the market response on a capital accumulation of certain individuals posed in the interaction network with many small agents.

We generalize [7], [5] and [1] using the non-constant couplings \( J_{ij} \) distributed according to constrained multimodal distribution, that we will explain now. It is also possible to take the Gaussian distribution \(^8\). In our model \( J_{ij} \) can be also negative and those negative values are giving rise also to the possibility of local non-complementarity between the agents. This assumption can be extended in different ways regarding the locality and intensity of the interactions. Even if \( J_{ij} \) can be negative, we keep them mainly positive constraining the multimodal distribution: the total coupling between an agent and his nearest neighbors is (locally) constrained in the usual quadratic lattice sense to be mainly positive, i.e. one agent takes, in average, positively correlated actions with his surrounding. More explicitly, on the quadratic lattice at least two of four \( J_{ij} \) will be positive. We used the nearest neighbors, but some inputs from econometrics, theoretical physics and graph

\[^8\text{This is inspired by some models in statistical physics as well as by fundamental theorems from the theory of probability.}\]
theory about the information propagation between the agents can be used to model much more realistically the grouping of individuals. For example, random graphs are one of the possible plausible ways to make conglomerates of agents; used in [15] gave rise to fat tails in returns distribution as observed for the high frequency market data.

Another important generalization is introducing of the information asymmetry (for the extensive empirical study see [17]). In this model, mathematically we incorporated just the information flow (i.e. influence) asymmetry by taking interactions $J_{ij}$ not symmetric anymore, i.e. $J_{ij} = J_{ji}$. We have simply that if an agent A is influencing a lot on an agent B, this does not mean that B is influencing in the same or similar measure on A. This can be in correlation with the richness of an agent: the richer the agent is (in our model rich agents are only those that can buy or sell $m$ units of asset), the more he influences the agents he is in contact with. So, we have that coupling $J_{ij}$ (the influence of agent i on the agent j) is bigger than $J_{ji}$ (the influence of agent j on the agent i) and, consequently, the interaction matrix $J_{ij}$ is not symmetric. We should expect also that the number of agents that are following certain rich agent behavior is bigger than the number of the agents that are in contact and tend to follow some smaller agents behavior. This property is currently not available in the program, but will be introduced in the future versions. The distribution is polymodal as $J_{ij}$ can take values in a finite set of values. Currently we are using: 0 – for agents that are not interacting, $\pm 1$ for a pair of small or a pair of big investors, $\pm 1$ is the influence of a small investor on a big investor, while $\pm l$, $l > 1$, measure the big to small investor influence, where $l$ is the parameter of the model.

4.1 Price Formation Rules

As proposed by many authors (see, for example, the extended discussion in [1]), on the base of the market equilibrium equations, we write for the price process, $P(t)$

$$\ln P(t + 1) - \ln P(t) = \frac{1}{\lambda} \sum_{i=1}^{N} s_i(t). \quad (7)$$

Here $\lambda$ is measuring liquidity of the market, or market depth (see [8] and [15]). Essentially, market depth is measuring the demand excess necessary to push the price by one unit.

The probabilities for large price changes are not as small as in the Gaussian distribution case. This should come out from the local properties of
the agents interactions. Models that produce heavy tails in the distribution of stock price variations has as the fundamental ingredient a random structure of the traders communication network. Price returns are distributed respecting the exponentially truncated powerlow and those distributions are observed especially in the high frequency financial market data for almost all assets.

5 Preliminary Results and Conclusions; Further Developments

We analyze the effects on measured quantities induced by the change of the percent of the big and small investors, the level of asymmetry and localization of the interactions and other model parameters.

Here we present three cases of the obtained price time series (in effect, their log returns) in which a discreet level of volatility clustering is present. We present here the models in which we used parameters \(a = 0\) and \(b = 0\).

The first plot, see figure 1, is the result of the our standard model, described in the Section 4, where the interactions of agents are realized as not symmetric. Agents are divided as big and small investors and a percentage can be varied easily and sistematically in future. The percent of big investors is taken to be 0.2 for all the cases. The parameter \(m\) (maximal number of bought or sold assets by big agents) of the model is taken to be \(l = 5\), while the parameter \(l = 10\).

The second plot, figure 2, corresponds to the asymmetric interactions and agents that are constrained to follow just mainly (not every single neighbour, as described in Section 4) the behaviour of their neighbours. The parameter \(l\) (maximal interaction) of the model is taken to be \(l = 5\), while \(m = 5\). In this model the interactions among big and small investors can not be zero, they can be just positive or negative (it is difficult that the decisions of big investors will not make any influence on the small investors decisions).

The third time series graphic, presented at 3, corresponds to the case where small investors always follow big investors decisions, i.e. interactions between them are always positive in the big to small direction. Interactions asymmetry and the possibility of local non-compatibility are still present for other pairs of agents; \(l = 1\), while \(m = 5\).

In this case the volatility clustering is stronger with respect to the previous two types of models.
In order to compare the synthetic time series with the real market data, we plan to measure next properties of the produced data: autocorrelation in price, autocorrelation in volatility (to quantify the volatility clustering), price-volatility correlations, the distribution of returns to see if they possess fat tails and Hurst exponent (or exponents, in the case the full multifractal structure of the obtained time series is needed). The final conclusions will be made after this analysis.

In the end, we will briefly list some of the possible model generalizations induced by the necessity to model the market structure in more details, that we plan to introduce in the future. The finite size scaling should be performed: the system characteristic functions and responses to external changes of the parameters should be traced as the size (linear dimensions in the square lattice case) of the market is left to grow. This will help to extrapolate the model behavior better versus real systems behavior. "Waves" of positive and negative information with correct time series properties could be introduced to simulate the quasi-equal-time receiving of the information; for example about the performances of the other financial markets around the world. This could imply a kind of stochastic resonance mechanism. Agents will be differently informed: some agents could receive and use certain external information or information about the other agents’ decisions, while the others not. Also, more financial products will be introduced.
Figure 2:

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Figure 3:


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