Coordination, intermittency and trends in generalized minority games

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Abstract

The Minority Game framework was recently generalized to account for the possibility that agents adapt not only through strategy selection but also by diversifying their response according to the kind of dynamical regime, or the risk, they perceive. Here we study the effects of this mechanism in different information structures. We show that both the stationary macroscopic properties and the dynamical features depend strongly on whether the information supplied to the system is exogenous (\textquoteleft random\textquoteright) or endogenous (\textquoteleft real\textquoteright). In particular, in the latter case one observes that a small amount of herding tendency suffices to alter the collective behavior dramatically. In such cases, the dynamics is characterized by the creation and destruction of trends, accompanied by intermittent features like volatility clustering.

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1. Introduction

The Minority Game (MG) and related models allowed to elucidate many aspects of the critical behavior of systems of heterogeneous inductive agents, with special

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emphasis on financial markets, by addressing directly the interplay between microscopic behavior and macroscopic properties—including fluctuations, market impact, predictability and efficiency—in rather elementary and often exactly solvable settings [1–3]. Besides providing a surprisingly rich description of the phase structure of an idealized speculative market [4], MG-based models are also able to reproduce, to some extent, the empirical price statistics (the so-called ‘stylized facts’ [5]) of financial markets [6–8]. However, the emergence of the peculiar dynamical regimes characterizing real systems, with intermittent fluctuation outbursts (volatility clustering) and the formation and destruction of trends, is only partially captured by standard MGs. This is in part due to the fact that the critical window where empirical phenomenology is observed typically shrinks as the system size increases, indicating that stylized facts would require a significant amount of fine tuning of the parameters and ultimately marginal efficiency [6]. Moreover, the dynamics of these models usually displays a strong dependence on both the disorder sample and the initial conditions [9]. It would therefore be desirable to devise a simple microscopic mechanism that is able to generate a realistic dynamical structure in a robust way while preserving the fundamental physical content of MGs.

Several studies in financial market microstructure suggest that these peculiarities emerge from the coexistence of different types of traders, and point attention especially toward ‘fundamentalists’ and ‘chartists’ [10–17]. The former trade on the spread between the actual and the expected price (the ‘fundamental’) trying to profit from fluctuations; the latter look for trends in the price history and follow the expected drift. The common picture has it that in market phases dominated by fundamentalists the price follows the fundamental, whereas when chartist pressure prevails trends deviating from the fundamental (bubbles) occur. Moreover, switches between the two phases, during which agents revise their expectations over a certain time span, are possibly accompanied by activity outbursts.

The expectations of agents in MGs are implicitly encoded in the reinforcement term of the learning dynamics. They are usually fixed: agents behave as fundamentalists in MGs and as chartists in Majority Games [18–21]. To implement the mechanism discussed above therefore means to modify the learning process so as to allow agents to change their expectations, and hence their character, in time according to the market conditions they perceive. One possible way to do this was introduced in Ref. [22]. In a nutshell, the idea is that when price movements are small agents should try to look for profit opportunities in emerging trends while when large price changes occur they should perceive a greater risk as the market behaves more chaotically, and return to a more cautious conduct. The passage from one attitude to the other is provided by a parameter that tunes the agents’ risk-sensitivity—or the trade-off between expected profit and perceived risk—so that when price movements exceed a given threshold the risk perceived by agents is large and they play an MG (that is, they select strategies that effectively try to follow the fundamental) preferentially. When coupled to a random external information structure, this mechanism was shown to lead to the formation of ‘heavy tails’ in the distribution of returns in a large ‘critical’ window where a crossover from a fundamentalist-dominated to a trend-followers-dominated market takes place.
Here we discuss this setting further by considering a somewhat simpler but more transparent model in order to address the role of the information structure, particularly in the system dynamics. This is an especially crucial point in analyzing the interaction between trend-followers and contrarians, since trend-following behavior is expected to introduce a strong bias in the endogenous information dynamics. We will indeed see that a small trend-following attitude is sufficient to drastically change both the typical stationary-state macroscopic properties and the single-sample dynamical properties with respect to the random information case. We will proceed by adding increasingly complex information structures to the simplest possible model (without information), which is introduced in Section 2. In Section 3 the case of random external information is considered while in Section 4 we address the case of ‘real’ endogenous information. Finally, we formulate some concluding remarks in Section 5.

2. The simplest model (without information)

Let us consider the following setup. Each of \( N \) agents \( (i = 1, \ldots, N) \) must decide whether to buy \( (a_i(t) = 1) \) or sell \( (a_i(t) = -1) \) at each time step \( t = 0, 1, \ldots \). The success of agent \( i \) at time \( t \) is measured by the payoff function

\[
\pi_i(t) = a_i(t)A(t)f[A(t)],
\]

where \( A(t) = \sum_i a_i(t) \) is the aggregate bid (or ‘excess demand’, which serves as a proxy for price movements). The function \( f \) encodes the type of game being played and, implicitly, the agents’ expectations. If \( f \) is a constant, agents are either playing a Majority Game (for \( f > 0, \pi_i > 0 \) if \( i \) acts according to the majority) or a MG (for \( f < 0, \pi_i > 0 \) if \( i \) acts according to the minority). Agents thus behave as trend-followers in the former case and as fundamentalists in the latter case. We wish to address a more general case in which agents are able to modify their character depending on the size of market fluctuations. We assume that they behave as trend-followers (resp. fundamentalists) when price movements are small (resp. large) and the perceived risk is small (resp. large). The simplest function for our scope is perhaps

\[
f(x) = \chi(|x| < L) - \chi(|x| > L),
\]

where \( \chi(B) = 1 \) if \( B \) is true (and 0 otherwise) and \( L \) is a threshold, so that when \( |A(t)| < L \) agents perceive the game as a Majority Game, whereas for \( |A(t)| > L \) they revert to a MG. For the sake of simplicity, in what follows we assume that \( L = O(N) \).

We imagine that in order to make their decisions agents employ the following iterated probabilistic rule \( (a \in \{-1, 1\}) \):

\[
\text{Prob}[a_i(t) = a] = \frac{e^{aU_i(t)}}{2 \cosh U_i(t)},
\]

where \( U_i(t) \) is some function of the state of the system and \( a \) is the action of agent \( i \) at time \( t \). This rule allows for a smooth transition between trend-following and fundamentalist behavior depending on the size of market fluctuations and the perceived risk.
where \( \Gamma > 0 \) is a constant (the ‘learning rate’ of agents) and \( U_i(0) \) are independent, identically distributed quenched random variables with zero mean and variance 1 for \( i = 1, \ldots, N \), and consider the properties of the steady state of the dynamics. We will see that, in spite of its extreme simplicity, this model is sufficient to capture one of the key features induced by this scheme.

Let us see, for a start, the behavior of the time average of the excess demand \( \langle A \rangle \) (where the average is taken in the steady state of the dynamics). In a Majority Game (\( f(x) = 1 \) or \( L \geq N \)) one finds \( \langle A \rangle \neq 0 \), implying that one of the two possible actions is systematically preferred (a trend), while \( \langle A \rangle = 0 \) in a pure MG (\( f(x) = -1 \) or \( L = 0 \)). Intuition suggests that for a sufficiently small \( L \) the latter scenario will dominate, since agents will be extremely risk-sensitive. On the other hand, for a sufficiently large \( L \) agents will be more risk-prone and a Majority-Game scenario is expected. The crossover between these two regimes is displayed in Fig. 1. One observes that for small \( L \) trends are formed (as in a Majority Game) even for small values of \( L \), whereas for large \( L \) the typical excess demand returns to zero (as in a MG) for all \( L < N \). It is also instructive to look at the crossover in the second moment \( \langle A^2 \rangle \), recalling that in a Majority Game, \( \langle A^2 \rangle \) is of order \( N^2 \) for any \( \Gamma \), while in a pure MG (see also insets in Fig. 1) \( \langle A^2 \rangle \) is of order \( N \) for small \( \Gamma \) and becomes of order \( N^2 \) for large enough \( \Gamma \). Finally, a deeper insight may be obtained from the correlation function \( \langle A(t)A(t+1) \rangle \) (see Fig. 2). For intermediate \( \Gamma \) excess demands are

\[
U_i(t + 1) - U_i(t) = \Gamma A(t) f[A(t)]/N ,
\]
negatively correlated for low $L$ whereas the correlation becomes positive when $L$ increases, that is, as agents become less and less risk-sensitive. The former regime is characteristic of MGs, where price increments are one-step anti-correlated because agents effectively act so as to compensate price increments in one direction with subsequent increments in the other direction. The latter regime is instead typical of trends and Majority Games.

This coexistence of MG-like and of Majority-Game-like features is at odds with mixed models in which each agent is either a fundamentalist or a chartist and is not allowed to pass from one group to the other [18,23]. In that case, the macroscopic properties are essentially determined by the larger group (be it fundamentalists or trend-followers), whose expectations are fulfilled. Here, the risk-sensitivity determines which group of traders dominates and mixed phenomenology occurs in a range of values of the learning rate $G$. While the detailed physical picture becomes more articulate, this conclusion actually extends to models endowed with more complicated information structure.

3. Case of random external information

Let us move over to the usual MG setting, namely a system of $N$ agents who at each time step $t$ must formulate a binary bid (buy/sell) based on some public information pattern $\mu(t)$. One such pattern is given to agents at every time step; it is assumed that the number of possible patterns is $P$ and that $P$ scales linearly with $N$. The relevant control parameter is indeed the relative number of information patterns $\alpha = P/N$. In order to translate information into bids each agent disposes of $S$ strategies, each one predicting the outcomes $a_{ig} = \{a_{ig}^b\}$ ($i = 1, \ldots, N$; $g = 1, \ldots, S$;
\( \mu = 1, \ldots, P \), and aims at selecting, at each time step, the one that delivers him the highest expected profit. If we denote this optimal strategy as \( g_i(t) \), the agent’s bid at time \( t \) is then given by \( a_{gi}(t) \). As usual, we assume that strategies have quenched random components drawn independently from \([-1, 1]\) with equal probability. In this work, we focus on the case \( S = 2 \).

The single-agent dynamics is defined by the following rules:

\[
  g_i(t) = \arg \max_g U_{ig}(t),
\]

\[
  A(t) = \frac{1}{\sqrt{N}} \sum_i a_{gi}(t),
\]

\[
  U_{ig}(t + 1) - U_{ig}(t) = a_{gi}(t) A(t)f[A(t)],
\]

where each \( U_{ig} \), called the ‘score’ in MG jargon, measures the performance of strategy \( g \) of agent \( i \). At each time step, every agent chooses his best-performing strategy as the one with the highest score and formulates the corresponding bid. Subsequently, bids are aggregated into the excess demand \( A(t) \), which we have now normalized for future convenience, and scores are updated. It is assumed here that scores are initialized at time \( t = 0 \) in such a way that \( U_{i1}(0) - U_{i2}(0) = 0 \) for all \( i \) and \( g \) (‘unbiased’ or ‘flat’ initial conditions). We will not address here the important and subtle issues of whether and how the steady state changes when a non-zero initial bias is used.

In this section, the information structure is assumed to correspond to the random external case: \( \mu(t) \) is an integer drawn randomly and independently at each time step from \( \{1, \ldots, P\} \) with uniform probability [24]. In this case the model is Markovian and the information dynamics uniformly covers the state space \( \{1, \ldots, P\} \). As for \( f \), we can borrow the recipe employed in the previous section and set:

\[
  f(x) = \chi(|x| < \eta) - \chi(|x| > \eta),
\]

where \( \eta > 0 \) is a tunable constant (notice that \( A(t) \) is of order 1 in this case). Different choices are also possible. For example, linearizing (6) one obtains

\[
  f(x) = \eta - |x|
\]

which is equally well suited for our purposes. The main difference from the piecewise constant case lies in the fact that payoffs (which, we recall, are proportional to \( A(t)f[A(t)] \)) are a non-linear function of the aggregate bid. Such non-linear choices, one of which was considered in Ref. [22], lead to the emergence of clear non-Gaussian statistics for \( A(t) \) at small values of \( \eta \). Here we wish to concentrate on the role of information structures and therefore we may restrict ourselves to the somewhat simpler form (6).

We will analyze the macroscopic properties in the steady state using \( \epsilon \) and \( \eta \) as control parameters. Our attention will be mostly focused on: (a) the volatility \( \sigma^2 = \langle A^2 \rangle \) measuring the magnitude of global fluctuations (notice that \( \langle A \rangle = 0 \) by
construction); (b) the ‘predictability’

\[ H = \frac{1}{P} \sum_{\mu} \langle A|\mu \rangle^2, \tag{8} \]

where \( \langle A|\mu \rangle \) stands for the time average of \( A \) conditioned on the occurrence of the pattern \( \mu \), quantifying the presence of exploitable information (if \( H \neq 0 \) the minority action can be statistically predicted on the basis of the information pattern alone at least for some \( \mu \)); and finally (c) the (normalized) one-step autocorrelation function \( D = \langle A(t)A(t+1) \rangle/\sigma^2 \), which indicates the dominating component of the market (if \( D > 0 \) returns are positively correlated and trend-followers dominate). Other observables of interest will be defined in due course.

Numerical results are shown in Fig. 3. One sees that the magnitude of fluctuations increases smoothly with \( \eta \), as one passes from an MG-like regime where \( \sigma^2 \) can be
smaller than the random-trading value 1, signaling a high degree of coordination among agents, to a Majority-Game-like regime (large $\eta$) where $\sigma^2 > 1$. Correspondingly, the system displays a transition between an unpredictable (or ‘symmetric’, low $z$) regime with $H = 0$ to a predictable (or ‘asymmetric’, high $z$) when $\eta$ is sufficiently small, similar to what occurs in the MG. Notice the critical point $\alpha_c$ below which $H = 0$ shifts (continuously) to smaller values as $\eta$ increases. Clearly, as the agents’ risk threshold grows their tendency to behave like trend-followers increases and, by herding, they produce more and more exploitable information ($H$ increases with $\eta$) even for smaller systems. Note that upon increasing $\eta$ further, one still observes a (sharp!) MG-like transition at low $\alpha$, but for high $\alpha$ the predictability tends to 1, as in the Majority Game. The change from one regime to the other at large $\alpha$ is apparently a threshold phenomenon in $\eta$ (see Fig. 4).\(^2\) The coexistence of the competing tendencies can also be seen from the fraction of ‘frozen’ agents (namely, agents for which $|U_{1i}(t) - U_{2i}(t)| \to \infty$ as $t \to \infty$, so that they use only one of their strategies in the stationary state), $\phi$. For small $\eta$, one finds a pure Minority Game; for intermediate $\eta$, $\phi$ jumps from the small-$z$ value of 0 (as in the MG) to the large-$z$ value of 1 (as in the Majority Game, where all agents ultimately freeze [21,23]). Notice that as $\eta$ decreases $\phi$ also decreases, which implies that when agents become more risk-sensitive it becomes more difficult for them to identify an optimal strategy.

Coming to the autocorrelation function $D$ (see Fig. 5), it shows the coexistence of Minority- and Majority-like features for intermediate values of $\eta$ once more. For small $\eta$, $D$ is negative and the dynamics is completely dominated by anticorrelations (i.e., by contrarians). As $\eta$ increases positive correlations appear for large $\alpha$, and the system is dominated by trend-followers for large $\eta$.

In summary, one can say that adding a certain risk-tendency at the microscopic level leads to a loss of global efficiency and that for intermediate values of $\eta$ MG-like features coexist with Majority-Game-like features, the former prevailing at low $\alpha$. This is again in sharp contrast with the scenario emerging from mixed Majority–Minority Games, where the expectations of the larger group (be it

\(^2\)A similar, although less sharp threshold phenomenon was found for the different although conceptually similar model of De Martino et al. [22].
fundamentalists or trend-followers) are fulfilled at all $\alpha$ and where the macroscopic properties are essentially a linear combination of those of the pure models [23].

It is important to notice that even when the competition between risk-aversion and profit-maximization is not too strong, for instance in the MG-like phase, the dynamics acquires several non-trivial traits. In Fig. 6 we report the results from a single realization of the time series of the excess demand $A(t)$ in correspondence with three different information patterns, and the time series of the ‘price’ $R(t) = \sum_{\ell \leq t} A(\ell)$, both for small $\alpha$, that is, where MG-like features are predominant. One can see that excess demand fluctuations at fixed $\mu$ behave intermittently: periods of high volatility are followed by periods of low volatility. At the same time, the profile of $R(t)$ shows the formation of well-defined trends.

### 4. Case of endogenous information

We now move to the case in which the information pattern $\mu(t)$ encodes in its binary representation the string of the last $m$ losing actions (the ‘history’) of the market sign $A(t - \ell)] (\ell = 1, \ldots, m)$ so that $P = 2^m$. The information dynamics in this case is deterministic and reads

$$
\mu(t + 1) = \begin{cases} 
[2\mu(t) + 1] \mod P & \text{if } A(t) > 0, \\
[2\mu(t)] \mod P & \text{if } A(t) < 0.
\end{cases}
$$

Trend-following behavior is expected to influence the macroscopic properties rather strongly, because of the bias that trends would impose on the resulting history.

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**Fig. 5.** Normalized one-step autocorrelation function $D$ as a function of $\alpha$ for different values of $\eta$. Simulations performed with $\alpha N^2 = 16,000$ and averages over 100 disorder samples per point.
dynamics. We therefore have to take into account the frequency $\rho(\mu)$ with which each string $\mu$ is generated in the steady state and modify the definition of the predictability (8) as

$$H = \sum_\mu \rho(\mu) (A|\mu|^2).$$

As a measure of the bias (or of the information content) of the frequency distribution one typically employs the entropy

$$S = -\sum_\mu \rho(\mu) \log P \rho(\mu)$$

which is normalized in such a way that for the random case discussed above, $\rho(\mu) = 1/P$ and $S = 1$.

Numerical results for the volatility $\sigma^2$ and the predictability $H$ (see Fig. 7) show that already for small values of $\eta$ (e.g. $\eta = 0.1$) the behavior deviates greatly from the one found in the case of exogenous information. In fact, while there is no evidence of a symmetric regime, fluctuations for small $\alpha$ are much smaller than in the MG regime. A detailed analysis of the volatility as a function of $\eta$ for small $\alpha$ (see Fig. 8) suggests that as soon as agents allow for a small amount of risk-proneness fluctuations decrease sharply, although their value may vary significantly from sample to sample. It is important to notice that since $N$ is odd, the minimum possible value of $|A(t)|$ is $\eta^* = 1/\sqrt{N}$ (which corresponds to $\eta^* \approx 0.043$ for the simulations of Fig. 8). For $\eta < \eta^*$ agents are always playing an MG which leads to large fluctuations in the low $\alpha$-phase, given the flat initial conditions. As soon as the risk threshold
Fig. 7. Volatility (top) and predictability (bottom) as a function of $a$ for different values of $\eta$. Simulations performed with $aN^2 = 30,000$, with averages over 200 disorder samples per point.

Fig. 8. Volatility as a function of $\eta$ for $a = 0.1$. Circles correspond to averages over 200 disorder samples; stars correspond to the values obtained in different samples. System with $N = 547$. The dashed vertical line marks the value $\eta^*$ corresponding to the minimum possible value of $|A(t)|$. 
exceeds $\eta^*$ and agents have a chance to herd fluctuations become significantly smaller. This remarkable effect, together with the fact that the predictability is small, indicates that relatively efficient states can be reached. As $\eta$ increases further $\sigma^2$ increases smoothly until it eventually reaches its standard Majority-Game value. (Note that the next smallest possible values of $|A(t)|$ are $3/\sqrt{N}$, $5/\sqrt{N}$ etc.; for $\eta$ in between these values the stationary volatility is roughly constant.) Thus, we see that the effects due to herding are much more pronounced in the presence of real histories with respect to the case of random information. Roughly speaking, one could say that a small amount of greediness at the microscopic level may turn out to have positive effects at the macroscopic level. The price to pay is the reduction of informational efficiency. This is yet another proof of the fact that the interplay between these two properties may be considerably subtle in these systems. However, larger risk thresholds lead to a (severe) loss of global efficiency. For large $\alpha$, instead, the model behaves as a pure MG. As $\eta$ increases, the pressure of trend-followers becomes stronger and the model acquires the character of a Majority Game more and more.

Analyzing the fraction of frozen agents (see Fig. 9) one sees that for small $\alpha$ almost all agents are frozen at the interesting values of $\eta$, so that even individual agents actually profit from a small greediness. Surprisingly, however, $\phi$ tends to the MG-like behavior when $\alpha$ increases. This is in striking contrast to the observations made for the model with exogenous information, in which MG-like features prevail at

![Fig. 9. Fraction of frozen agents (top) and normalized one-step autocorrelation function (bottom) as a function of $\alpha$ for different values of $\eta$. Simulations performed with $\alpha N^2 = 30,000$, with averages over 200 disorder samples per point.](image-url)
small $\alpha$ when coexisting with Majority-type of features. The results for the autocorrelation function $D$ confirm that indeed a small $\eta$ is sufficient to induce strong herding effects for small $\alpha$, at odds with the previous case.

Let us now look at the history dynamics. The entropy $S$ is reported in Fig. 10. First of all, for sufficiently small $\eta$ the scenario of a pure MG should be reproduced, where $S = 1$ for $\alpha < \alpha_c \simeq 0.34$ and $S < 1$ (slightly) for $\alpha < \alpha_c$. This is indeed the case, as can be seen even from Fig. 11, where we plot the steady-state distribution of history frequencies relative to the uniform case:

$$Q(f) = \frac{1}{P} \sum_{\mu} \delta[f - P \rho(\mu)]$$

(12)

(if $\rho(\mu) = 1/P$ for all $\mu$, $Q(f)$ is a delta-distribution at $f = 1$). As $\eta$ increases the entropy drops drastically for small $\alpha$ as herding trivializes the history dynamics. For large $\alpha$, again, the MG behavior is recovered. As $\eta$ increases and trend-following becomes more and more preferred, $S$ consistently tends to be much smaller than 1 as only a small fraction of histories are generated per sample. However, a look at the single sample behavior suffices to understand that the dynamics is much more complex than the entropy would and could tell (see Fig. 12). For small $\eta$ and small $\alpha$ one distinctly observes volatility clustering in the time series of $A(t)$. Periods of low volatility correspond to a trend, as shown by the distribution $Q(f)$ relative to those intervals. Here, the game has the character of a Majority Game and trend-followers dominate. Periods of high volatility correspond instead to a chaotic dynamics where the history frequency distribution is uniform. Here, the model behaves like a MG and fundamentalists dominate. The switch from one regime to the other is essentially driven by random events. Indeed, the duration of the activity outbursts, as well as that of trends, is arbitrary and may vary strongly from sample to sample. The
Fig. 11. Relative distribution of frequencies $Q(f)$ for $\eta = 0.01$ at $\alpha = 0.1$ (top) and $\alpha = 2$ (bottom). Simulations performed with $\alpha N^2 = 30,000$, with averages over 100 disorder samples per point.

Fig. 12. Single sample excess demand as a function of the time elapsed from equilibration for $\eta = 0.06$ (top) and $\alpha = 0.1$. Relative frequency distribution $Q(f)$ for the same realization in the different regimes (bottom). Inset: ‘price’ $R(t)$ as a function of time.
sharpness of the switch is instead simply due to the particular form of the function \( f \) we have chosen, as well as the value of \( \eta \). In fact, as one increases \( \eta \), the magnitude of fluctuations in the Majority-like periods increases and the different regimes tend to merge.

5. Conclusion

The simple microscopic mechanism introduced in our model, when coupled to real information, determines significant effects in the stationary macroscopic properties and produces realistic dynamical features such as volatility clustering. These results, together with the observations made in Ref. [22] on the statistics of price changes, show that generalized MGs reveal a remarkably rich and realistic behavior which has been only partially uncovered so far. By all means, we believe that the statistical mechanics of systems of interacting trend-followers and contrarians constitute an extremely challenging problem for physicists and are definitely worth more detailed investigations. Of course, an analytical solution would be welcome. To conclude we would like to indicate an issue that is, in our opinion, particularly interesting, namely observing how this scenario would change if the risk threshold were allowed to fluctuate in time. The most intuitive way to do this is perhaps to couple the threshold’s dynamics to the system performance. A possible microscopic mechanism could be the following. When \( \eta \) is large a high volatility is to be expected as agents are more likely to behave as trend-followers. As a consequence, they should likely reduce their threshold since the market is risky; however, for small \( \eta \) fundamentalists are expected to dominate and the game should acquire a Minority character. Hence the predictability will be smaller and there will be less profit opportunities. Agents may then decide to adopt a larger threshold to seek convenient speculations on a wider scale. If these two competing effects are appropriately described by an evolution equation for \( \eta \), the system should self-organize around an ‘optimal’ value of the risk threshold. For all practical purposes, the model discussed in this work assumes that such a time evolution takes place on time scales much longer than those over which trading occurs.

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