The Period of Financial Distress in Speculative Markets: Interacting Heterogeneous Agents and Financial Constraints

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Abstract: We investigate how stochastic asset price dynamics with herding, financial constraints, and variations in switching strategies in heterogeneous agents' decisions explain for the first time the presence of a period of financial distress (PFD) following the peak and preceding the crash of a bubble, documented by Kindleberger [2000, Appendix B] as common among most major historical speculative bubbles. Simulations show the PFD is due to agents' wealth distribution dynamics, selling because of financial constraints after the bubble's peak in relation to switching behavior of agents. An increase in switching tendency increases the length of the PFD and decreases bubble amplitude, while increasing strength of interaction between the agents increases bubble amplitude.

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I. Introduction

In the fourth edition of his magisterial Manias, Panics, and Crashes [2000], Charles Kindleberger has an appendix [B] that lists a series of famous speculative bubbles and crashes in world history. The list begins with the 1622 currency bubble of the Holy Roman Empire during the Thirty Years War and ends with the Asian and Russian crises of 1997-98. In his discussion of how speculative bubbles operate, drawn heavily on work of Hyman Minsky [1972, 1982], Kindleberger identifies a general pattern followed by most of them. There is an initial displacement of the fundamental that begins the bubble, although not all have a well defined such displacement. Later the bubble reaches a peak after a period of credit expansion and speculative euphoria. Then for most there is another date after the peak when there is the crash or crisis. Kindleberger calls the period in between these two dates the period of financial distress. Of the 46 bubbles listed in this appendix, Kindleberger identifies 36 as having such a period as indicated by having clearly distinct dates for a peak and a later crash, with a few others potentially having one.

One can argue with his list. Missing bubbles include the US silver price bubble that peaked and crashed in 1980 and the US NASDAQ bubble that peaked and crashed in March 2000, following the pattern set by the first two on his list, the Holy Roman Empire bubble and then the Dutch tulipmania that crashed suddenly on February 5, 1637 [Posthumus, 1929; Garber, 1989], with the last one on his list from 1997-98 also showing this pattern. Nevertheless, to the extent that Kindleberger’s list reasonably reflects historical patterns, it would appear that a solid majority of historically noteworthy speculative bubbles had such a period of financial distress, a period after the peak of the bubble in which the market declined somewhat gradually before it dropped more precipitously in a panic-driven crash. Even the most famous stock market crash (October 1929) followed a similar path: as figure 1 shows, it peaked in August before eventually crashing 2 months later.
To date there have been only a few theoretical models that have separated a peak from a crash, [De Long et al, 1990, Rosser, 1991, 1997]. One problem has been the widespread reluctance by economic theorists to accept the reality that potentially speculative markets have heterogeneous agents, reflecting a favoritism for representative agent models in which the agent in question has rational expectations. Indeed, under sufficiently strict conditions (a finite number of infinitely lived, risk-averse, rational agents, with common prior information and beliefs, trading a finite number of assets with real returns in discrete time periods) it can be shown that speculative bubbles are impossible [Tirole, 1982]. Influenced by the spectacular crashes in 1987 and 2000, economists have become increasingly willing to doubt the realistic applicability of such theorems to actual markets. DeLong et al [1991] showed that “noise traders” not only could survive, but even that some of them may outperform the supposedly rational fundamentalist traders in the market. Such arguments have opened the door to studies that emphasize the roles of heterogeneous interacting agents [Day and Huang, 1990; Arthur et al, 1997; Lux, 1998; Kaizoji, 2000, Chiarella et al, 2003, Bischi et al, 2005]. However, none of these
have demonstrated the kind of pattern described by Minsky and Kindleberger as the "period of financial distress."

In the next section we briefly discuss the literature on stock market crash. We shall then present a mean field model that has been used by Chiarella et al. [2003] for a larger set of heterogeneous agents who interact with each other, derived from work originally done by Brock [1993], Brock and Hommes [1997], and Brock and Durlauf [2001]. We introduce elements such as a wealth constraint and differing propensities to switch strategies into such a heterogeneous interacting agents model to study this period of financial distress. Section 4 concludes.

II. Bubbles, Crashes and Financial Distress

De Long et al [1990] were principally concerned with demonstrating the possibility of "rational speculation" in the presence of noise traders, with the rational speculators forecasting the future purchases by the noise traders and thereby being able to make money by buying in advance of them. In short, the rational speculators are the "insiders" and the noise traders are the "outsiders." Studying the period of financial distress is not the main focus as the outcomes in the model do not show such a phenomenon. Rather, what is seen is prices rising at a slower rate as the noise traders enter the market rather than falling as they do in a classic period of distress. In De Long et al., there is no period of financial distress modeled in the Minsky-Kindleberger sense. The trend chasing of the noise traders guarantees that the bubble will continue to rise, even with the rational speculators selling in that period. This outcome can be altered by introducing many more periods and a lag operator within a stochastically crashing bubble framework (Blanchard and Watson, 1982) as in Rosser [1991, Chap. 5; 1997]. Three cases can be seen of sudden crash at peak, gradual decline after peak, and a period of financial distress before the crash. However, this model involves strong assumptions, and the resulting measure of the parameter space in which the period of financial distress can be observed is small. Furthermore, the strong rationality assumption of the De Long et al [1990] model is
partly relaxed. The peculiar behavior of these models reflects both that the agents are of extreme types and the artificial nature of the modeling of the crash.

In the more general interacting agents model we consider in this paper, agents are allowed to be of more intermediate types, to have intermediate levels of relations with each other (tendencies to herd or chase trends) and varying degrees of willingness to change their behavior, all in the context of operating within a wealth constraint. The tendency to chase trends and the willingness to change behavior have obvious links with parameters in the model of Rosser. With this more flexible and realistic approach we are able to understand the situations in which the historically common period of financial distress phenomenon can arise.

Historical discussions of the most spectacular of the early bubbles, the closely intertwined Mississippi bubble of 1719-20 in France and the South Sea bubble of 1720 in Britain, show a standard pattern [Bagehot, 1873; Oudard, 1928; Wilson, 1949; Carswell, 1960; Neal, 1990]. Common to all these discussions are two groups of agents, a smart group of “insiders,” who buy into the bubble early and who get out early, usually near the peak, and a less well-informed (or intelligent or experienced) group of “outsiders” who do not get out in time. These are the agents who continue to prop the bubble up during the period of distress as the wiser insiders are selling out. The crash comes when this group of outsiders, for whatever reason, finally panic and sell. In discussing the British South Sea bubble, Wilson [1949, p. 202] characterizes this outsider group as including “spinsters, theologians, admirals, civil servants, merchants, professional speculators, and the inevitable widows and orphans.” An important factor in many of the actual crashes, noted especially for the 1929 stock market crash by Minsky, Kindleberger, and also Galbraith [1954], is that investors can encounter wealth constraints, especially if they have borrowed on margin to buy assets. The crash itself can be exacerbated by a mounting series of margin calls that force investors to sell to meet the calls, thereby pushing the price further down and triggering more such calls.

Efforts have been made to model speculative bubbles using the interactions of such insiders and outsiders [Baumol, 1957; Telser, 1959: Farrell, 1966], although without showing such a period of distress. Others have simply shown interactions between fundamentalists who do not participate in the bubble and trend-chasing
chartists who do, but without subdividing them [Zeeman, 1974]. However, all these efforts fell into disrepute as the rational expectations revolution gathered steam during the 1970s. More recent efforts to model periods of financial distress have been carried out using insider-outsider models in models of financial crises in emerging market foreign exchange rates, although without showing a period of declining currency value prior to a full crash, or “sudden stop” [Calvo and Mendoza, 2000].

Rosser [1991, Chap. 5] provides three canonical patterns for bubbles and crashes, drawing on the discussion by Kindleberger. The first is that of the accelerating bubble that is followed by a sudden crash, much like that of the Dutch tulipmania on February 5, 1637. This is depicted in Figure 2. Most of the models of rational agents in bubbles tend to follow this pattern [Blanchard and Watson, 1982]. Another is that of a bubble that decays more gradually after rising, such as in France in 1866 or in Britain in 1873 and 1907. This is depicted in Figure 3. It is often argued that many bubbles follow an intermediate path between these two, with a decline that is not a discontinuous crash, but that asymmetrically declines more rapidly than it increased. This has been studied using heterogeneous interacting agents models [Chiarella et al, 2003]. Finally there is the pattern we are studying in this paper, the historically most common pattern according to Kindleberger, that of the bubble that exhibits a period of financial distress after the peak but prior to the crash. This is depicted in Figure 4.

Figure 2
III. The Model

In this section we will describe a model able to explain financial bubbles characterized by a period of distress. The model follows the framework described in Bischi et al. [forthcoming] introducing agents' financial constraints. We consider a population of investors facing a binary choice problem. The agents who decide to trade have to choose a strategy \( w_i(t) \in \{-1;+1\} \), where \(-1\) stands for «willing to sell», while \(+1\) stands for «willing to buy» a unit of a given share. We do not model explicitly an optimal portfolio problem; rather the trade decisions \( w_i(t) \) have to be interpreted as the marginal adjustment the agents make as they try to be advantaged by profit opportunities arising due to continuous trading information diffusion. In the following we use the assumptions:

- (i) There exist 2 assets: a risk free asset with a constant real return on investment \( r \) and a risky asset with price \( P(t) \) that pays a dividend, say every year, supposed to follow a stationary stochastic process \( d(t) \).
- (ii) Agents observe past prices, the relative excess demand, \( w(t) = N(t)^{-1} \sum w_i(t) \), the real interest rate, \( r \), and have rational expectations about the dividend process (their expected value is equal to \( d \), the mean of the process). Therefore the fundamental solution of the risky asset price is the ratio \( F = \frac{d}{r} \). The information set of the agent is the union of his/her private characteristics, say the set \( \Omega_i(t) \), and the public information set \( \Omega(t) = \{r,d,w(t-1), w(t-2),..., P(t-1),P(t-2),...\} \).
- (iii) In order to take their buy/ sell decision, the agents evaluate an expected benefit function \( V_i(t) \), that will depend on their prior believes on the price that will prevail in the market. We assume that the agents engage in rational herd behavior, i.e., they expect that \( V_i(t) \) will be positively related with the other agents' buy/ sell decisions.
- (iv) Price dynamics is assumed to follow the difference equation

\[
p(t+1) - p(t) = f(w(t)) + \sigma_i z_i(t+1),
\]
where \( p(t) \) is the logarithm of \( P(t) \), and \( f(w(t)) \) is a deterministic term, that measures the influence of excess demand on current price variations, with properties: \( f(0) = 0, f'(w(t)) > 0 \). The stochastic component of price dynamics is captured by \( \sigma_i z_i(t+1) \) where \( z_i(t+1) \) is a NID(0,1) process so that \( \sigma_i \) determines the variance of the shocks. Note that, when the excess demand is zero, both the conditional and the unconditional distribution of price changes follow a Gaussian process with zero mean and variance \( \sigma_i^2 \). With an out-of-equilibrium dynamics \( w \neq 0 \) the conditional distribution will have a different mean while remaining Gaussian by definition, whereas the unconditional distribution may not belong to the normal distributions family [see Leombruni et al. 2003].

- (v) Agents have homogeneous expectations on the relative excess demand at period \( t \), say \( w(t)^e \). Following Brock and Durlauf [2001b], agents' static expectations with respect to their information set are assumed; i.e., \( w(t)^e = w(t-1) \).

The agent's choice \( w_i(t) \) is modeled as a binary random variable that describes, from the point of view of the modeler, the choice of agent-\( i \) at time \( t \) between the two strategies. In other terms, the random variable \( w_i(t) \) gives the probability distribution of agents' decisions conditionally on his/ her expectations. With perfect information and perfect market efficiency, the relevant statistic to compute would be the ratio between the expected value of the fundamental solution of price dynamics, \( \bar{p}(t) \), and the actual price \( P(t) \), that measures the expected rate of profit (loss) when the price reaches the fundamental. Then, the relevant statistic, in logs, is \( \bar{p}(t) - p(t) \).

However, we assume that the agents do not consider the latter a sufficient statistic in order to analyze the implications of herding on the assets' price dynamics. The rationale of this imitative behavior is that the agents try to extrapolate/ exploit from the observed choice of the others the piece of information he/ she is lacking.
Following Bischi et al. [forthcoming], a convenient way to model the herd component in the behavior of the investors, is by means of a binary choice framework with interaction. Namely, we assume that the non-financially constrained expected benefit function for the strategy \( w_i(t) \) is

\[
U_i(t + 1, w_i(t)) = (\bar{p}(t) - p(t))w_i(t) + J(t)w_i(t)w(t)^\gamma + \varepsilon_i(t + 1, w_i(t))
\]

The equation above is a standard assumption in the social interaction literature (see Brock and Durlauf, [2001a,b]). It implies that the utility, or benefit function, is affected by three additive components. The first component gives the private benefit in choosing strategy \( w_i(t) \). The second one is an interaction term (proportional spillovers) measuring the benefit of that choice in a situation where the expected average choice is \( w(t)^\gamma \). Finally, the last term introduces, stochastically and from the point of view of the modeler, idiosyncratic factors and private information, \( \Omega_i(t) \), affecting agents decisions.

To be precise, The first term of the right hand side is the benefit of the strategy «to buy one unit of share» \( (w_i(t) = 1) \) or «to sell one unit of share» \( (w_i(t) = -1) \) in case the agent would consider only the fundamental solution of price. The second term captures the positive spillover the agent-\( i \) expects in following the others expected choices. It captures the interaction among investors, in the form of a proportional spillover \( Jw_i(t)w(t)^\gamma \). In other words, the benefit expected by agent \( i \) depends on his/her expectation about the average choice of the market, \( w(t)^\gamma \). The positive function \( J(t) \) measures the weight given to the choices of other agents. We assume that the strength of the interaction is endogenous: when the price is far from individual expectations, the agent is less confident in the mood of the market, and gives a smaller weight to the interaction term of his/her forecast. Therefore, \( J(t) = J(p(t)) \) is a decreasing function of \( |\bar{p}(t) - p(t)| \) with \( J(\bar{p}(t)) < \infty \). This assumption is made for two reasons: 1) research on herding in US equity markets indicates that herding does not take place during periods of market stress or high price volatility – during periods of extreme market movements [Christie and Huang,
1995; Gleason, Mathur and Peterson, 2003]; 2) to show that a sufficient level of herding may generate asset price fluctuations, that do not depend on changes in the fundamental, even when there is an adjustment mechanism in herding behaviors; i.e., when the herding component, in the agent’s decision process, tends to disappear when the asset price go far from the fundamental solution.

The discrete choice literature calls the term \((\bar{p}(t) - p(t))w_i(t) + J(t)w_i(t)w(t)\)' the «deterministic component of the expected benefit function». Instead, the third term, \(e_i(t+1, w_i(t))\), represents the random variables that may have different distributions under the two choices. As said above, they capture agents’ unknown (to the modeler) features, the key assumption of this model being the presence of a financial constraint in agents’ decision process. In other terms, agents start the continuous trading with a given initial wealth and remain in the market only if their losses are not to high. Formally, agent’s benefit function with liquidity constrains may be expressed with the following equation

\[
V_i(t+1, w_i(t)) = \begin{cases} 
U_i(t+1, w_i(t)) & W > \theta W_0 \\
-\infty & W \leq \theta W_0 
\end{cases}
\]

with \(0 < \theta < 1\) measuring the fraction of the initial wealth below witch the agent sells with probability 1. In the following section, agents’ stochastic decisions regarding whether to buy or to sell could be described by the probability that the benefit function will yield a higher benefit than the other choice, that is

\[
Prob(w_i) = Prob(V_{i+1}(w_i) > V_{i+1}(-w_i)) .
\]

So far, we did not consider any dynamics in the priors about the expected price. Actually, when an investor “follows the herd” because of the (assumed) presence of information asymmetries, he/ she should coherently revise his/ her priors. For instance, if he/ she follows the herd during a bull market, we should expect that he/ she will contextually increase his/ her prior on the fundamental. More generally, we can model the priors revision assuming that the agents adjust their
private expectations comparing them with the public information that is currently mirrored in the price level. That is, we can assume the following adaptive adjustment mechanism for the priors on the expected price:

\[
\bar{p}(t+1) = \bar{p}(t) - \rho \left( \bar{p}(t) - p(t) \right) + \sigma \cdot z_2(t+1)
\]

where \( \rho \in [0,1] \) is a measure of the adaptive speed of adjustment. The adaptive mechanism given in the equation above can be described by saying that the new expected price is a convex combination of the previous expected price and the previous realized price (\( \rho \) being the relative weight of the realized price) plus a stochastic component \( \sigma \cdot z_2(t+1) \) where \( z_2(t+1) \) is a NID(0,1) process so that \( \sigma \) determines the variance of the shocks.

**IV. Stability Analysis in the Deterministic Case without Financial Constraint**

To get an insight of the main features of the model, in the following we first describe the deterministic version (random variables in the model are setted to zero), without liquidity constraints, analyzed in Bischi et al [forthcoming] where it is proved that, when the number of agents becomes large, the price, the expected price, and the relative excess demand follows the three-dimensional dynamical system

\[
\begin{align*}
T_3: & \quad w(t+1) = \tanh \left( \beta (\bar{p}(t) - p(t) + w(t) J(\bar{p}(t) - p(t))) \right) \\
& \quad p(t+1) = p(t) + f(w(t)) \\
& \quad \bar{p}(t+1) = \bar{p}(t) - \rho (\bar{p}(t) - p(t))
\end{align*}
\]

Introducing the dynamic variable \( q(t) \), defined as

\[
q(t) = p(t) - \bar{p}(t),
\]

that represents, at each time period \( t \), the difference between the current price and the expected price we obtain a dynamical system in the variables \( (w(t), q(t), \bar{p}(t)) \) expressed by
\[
\begin{align*}
T_3: & \quad \begin{cases}
w(t+1) = \tanh(\beta(-q(t) + w(t)J(|q(t)|))) \\
q(t+1) = (1 - \rho)q(t) + f(w(t)) \\
\bar{p}(t+1) = \bar{p}(t) + \rho q(t)
\end{cases}
\end{align*}
\] (8)

Of course, this model is equivalent to the model (6), in the sense that the two models are topologically conjugate, but the model in the form (8) reveals a property that simplifies its mathematical analysis: the first and the second dynamic equations in (8) only involve the dynamic variables \( w \) and \( q \), i.e. they represent an autonomous two-dimensional dynamical system represented by the iteration of a two-dimensional map, say \( T_3 : (w(t), q(t)) \to (w(t+1), q(t+1)) \). This means that the dynamics of \( w(t) \) and \( q(t) \) are not influenced by \( \bar{p}(t) \), while the time evolution of \( \bar{p}(t) \) is influenced by the dynamics of the two-dimensional system governed by \( T_3(w, q) \) due to the presence of \( q(t) \) in the third dynamic equation. The expected price \( \bar{p}(t) \) can be easily obtained from the time series \( \{ q(\tau), \tau = 0, \ldots, t-1 \} \) by the closed form

\[
\bar{p}(t) = \bar{p}(0) + \rho \sum_{\tau=0}^{t-1} q(\tau).
\] (9)

That is, starting from an initial expected price \( \bar{p}(0) \), the expected price at time \( t \) is determined by adding the algebraic sum of \( q = p - \bar{p} \) observed in the past, modulated by the parameter \( \rho \) (a higher value of \( \rho \) determines a stronger influence of the past history on \( \bar{p}(t) \), whereas \( \rho \to 0 \) implies no changes of the expected price, i.e. \( \bar{p}(t) = \bar{p}(0) \, \forall t \geq 0 \)). However, even if one knows the kind of behavior of \( q(t) \), the analysis of the corresponding behavior of \( \bar{p}(t) \), given by (4), is not straightforward. First of all, a necessary condition for the convergence of \( \bar{p}(t) \) to a finite stationary asymptotic value is that \( q(t) \) goes to 0 as \( t \) increases, i.e. the discrepancy between the expected and the realized price vanishes. However, even if \( p(t) \) converges, its limiting value \( \bar{p}_L(t) \) is strongly influenced by the transient part of the sequence \( \{ q(t) \} \), determined by the driving system, before it enters a neighborhood of 0. This implies that \( \bar{p}_L(t) \) is highly path dependent, because any change of the initial
condition \((w(0), q(0))\) of the driving system causes a change of the asymptotic value 
\(\bar{p}_L(t)\) of the expected price (and consequently of the current price, being 
\(q(t) = p(t) - \bar{p}(t)\) convergent to zero). This means that any exogenous shocks or other 
historical accidents are «remembered» by the system, i.e. their effects are not 
canceled by the endogenous dynamics resulting in a long memory property for the 
financial time series.

\[ \frac{1}{\beta}, \frac{(1+\rho)/\beta}{\rho^2/\beta}, \frac{(1-\rho)/\beta}{1/\beta} \]

\[ J'(0) \]

Figure 5: Stability domain of the equilibrium \((0,0)\) of the 
deterministic two-dimensional driving system \(T_2\).

The stability analysis of the autonomous two-dimensional system \((T_2)\) and the 
expected price equation reveals some different kinds of dynamics of the variables 
\(w(t), q(t)\) and \(\bar{p}(t)\):
• **(a)** convergence to the steady state $E_0 = (0, 0)$ (the whole shadow area in figure 5), and such convergence may be oscillatory (the darker part) or monotonic;

• **(b)** a situation of bistability, with stable equilibria characterized by positive and negative coordinates respectively, whose basins of attraction are separated by the stable set of the saddle point $E_0$; via a supercritical pitchfork bifurcation.

• **(c)** periodic or quasi-periodic oscillations along closed invariant curves located around the unstable focus $E_0$; via supercritical Neimark-Hopf bifurcation.

• **(d)** However, even if $p(t)$ converges, its limiting value $\bar{p}_L(t)$ is strongly influenced by the transient part of the sequence $\{q(t)\}$, determined by the driving system, before it enters a neighborhood of $0$. This implies that $\bar{p}_L(t)$ is highly path dependent, because any change of the initial condition $(w(0), q(0))$ of the driving system causes a change of the asymptotic value $\bar{p}_L(t)$ of the expected price (and consequently of the current price, being $q(t) = p(t) - \bar{p}(t)$ convergent to zero). This means that any exogenous shocks or other historical accidents are «remembered» by the system, i.e. their effects are not canceled by the endogenous dynamics.

### V. Simulation results

In this section the complete model is analyzed by simulations that are implemented assuming an annual dividend with mean $d=1$ and $r = 0.1$, therefore the fundamental solution $F$ is equal to 10. We specify a linear in log price dynamics with $f(w_t) = k w_t$, that is

\[ p(t+1) - p(t) = kw(t) + \sigma \gamma z_i(t+1), \]
where $k = 0.4$ and the standard deviation $\sigma_1 = 2 \times 10^{-6}$, whereas $\sigma_2 = 4 \times 10^{-2}$. The initial log expected price is $\ln(10.1)$ whereas $p_0 = \ln(10)$. The $J$ and $\beta$ was set equal to respectively 0.5 and 0.1. For every agent $i = 1 \ldots 100$ the buy/sell decision is made accordingly to the probability measure described in section 3. The following realized profit is computed:

$$\Pi_i(t) = w_i(t)[p(t+1) + y(t) - p(t)] - c$$

where $y(t)$ is the part of the dividend attributed to the time interval $(t, t+1)$. During the simulations were used the value $d/36000 = 1/36000$. Finally, $c$ is a transaction cost assumed constant and equal to 0.001. The simulations were started with the initial wealth of every agent, $W_0$, equal to 1000 dollars. In the simulations with financial constraint we set $\theta = 0.7$. In other terms, we assume that when agents loose 30% of their initial wealth they sell with probability 1, exit the market, and are replaced by new ones with initial wealth equal to 1000 dollars.

We observe two kind of phenomena: 1) bubbles are of the second type like the one stylized in figure 6. A numerical example (figure 4) show that even using a bigger value of $J$, compared to the other simulations, the time series bubble look like figure 6. The black line is the evolution of the expected price whereas the gray line represent the asset price dynamics.
In other terms, even though fluctuations may be asymmetric and may show a change of slope during downfall, they do exhibit a crash preceded by a period of financial distress; 2) with this parameters the distribution of wealth, $W$, is changing in location and shape, i.e. the mean decreasing and the variance increasing but, because of the not-binding financial constraint, this have no effect in asset price dynamics.

If we allow the financial constraint to bind we may observe simulations like figure 7 in which a period of financial distress appears.
Because of the path dependence property that characterized the model, we construct the figure using the same parameters and the same random seed. The gray (black) line describe the evolution without (with) financial constraint.

Performing a Monte Carlo of 200 simulations with 100 agents, each of 1000 run with parameters as above, but changing the random seed in the random number generator we get 84 crashes preceded by a PFD. We consider a PFD only if its duration is at least 10 periods. During simulations the maximum (minimum) value of the duration of the PFD was 158 (11); the mean value of duration was 61.23 with standard deviation 29.12 (tab. 1, first row).

<table>
<thead>
<tr>
<th>β</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>stand.dev.</th>
<th>Reduction in amplitude (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>11</td>
<td>158</td>
<td>61.23</td>
<td>29.12</td>
<td>22.71</td>
</tr>
<tr>
<td>0.7</td>
<td>19</td>
<td>202</td>
<td>72.69</td>
<td>37.64</td>
<td></td>
</tr>
</tbody>
</table>

Tab. 1: period of financial distress statistics
Since we do not obtain such phenomena without the introduction of the financial constraint the following proposition can be asserted

- **Proposition 1**: bubbles characterized by a PFD are generated when the agents are financially constrained.

The explanation of type 3 bubbles is evident when looking at the evolution of the wealth distribution of agents (figure 8). The 3 densities (from right to left) describe the situation at the beginning (t=20), then after 200 periods and the last is the wealth distribution just before the crash (t=360) where it is possible to see that a large tail of the distribution is at or below the threshold $0.7 \times 1000 = 700$. In other terms,

- **Proposition 2**: the PFD is the time necessary, after the maximum of the bubble, in order to have a sufficient mass of agents needing to sell because of financial constraints.

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Figure 8: wealth distribution dynamics
An increase of the parameter $\beta$ from 0.1 to 0.7 produces an increase in the mean and the variance of the PFD distribution (tab.1, second row). An example of this result, obtained with different choice of the parameter, is shown in figure 9. Tab. 1 (second row) summarize a Monte Carlo simulation with parameters as above but $\beta$ was increased from 0.1 to 0.7. During simulations the maximum (minimum) value of the duration of the PFD was 202 (19); the mean value of duration was 72.69 with standard deviation 37.64. Furthermore, in every run on the second set ($\beta = 0.7$) the amplitude of the bubble decreases. The average reduction is 22.71%. This value was computed evaluating the reduction in percentage of the maximum preceding the crash. Summarizing the results, the following proposition can be asserted.

- **Proposition 3**: An increase of the parameter $\beta$ increases the PFD and decreases the amplitude of the bubble.
Figure 9: an increase in $\beta$. The two time series share the same random numbers and same parameters but $\beta$. In the gray time series $\beta = 0.1$; in the black time series $\beta = 0.7$.

Figure 10: increase in $J$. The two time series share the same random numbers and same parameters but $J$. In the gray time series $J = 0.5$; in the black time series $J = 3$. 
Finally, a third set of simulations is performed with the same random numbers of the first one except the parameter $J$ that increases from 0.5 to 3. Every run results in a greater amplitude of the bubble but no significant change in duration. An example of such simulation is shown in figure 10. Tab. 2 summarize the results of the third simulation (second row) compared to the first one (first row),

<table>
<thead>
<tr>
<th>J</th>
<th>Mean (PFD)</th>
<th>Stand.dev. (PFD)</th>
<th>Increase in amplitude (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>J = 0.5</td>
<td>61.23</td>
<td>29.12</td>
<td>56.87</td>
</tr>
<tr>
<td>J = 3.0</td>
<td>62.18</td>
<td>30.23</td>
<td></td>
</tr>
</tbody>
</table>

Tab. 2: the effect of an increase in $J$ from 0.5 to 3. The last column show the average percentage increase at the maximum of the bubble.

The second set of simulations show a 56.87% average increase in the mean value of bubble amplitude.

- **Proposition 4**: An increase of the parameter $J$ increases the amplitude of the bubble.

### VI. Concluding Remarks

We have considered how introducing a financial constraint into a framework characterized by herding and switching of priors about the fundamentals by heterogeneous investors can explain the appearance of a period of financial distress between the peak and the crash of a speculative bubble. The incompleteness of the agents' information set is sufficient for non fundamentalist analysis in the agents' decision process modeled in a binary choice setting with interacting agents.

The analytic study of the deterministic case without liquidity constraint is analyzed in Bischi et al. [forthcoming]. The model shows for financial markets convergence to the steady state with excess demand equal to zero and the asset price equal to the fundamental solution, with such convergence oscillatory or monotonic. Increasing the sensitivity of the risky asset price to the relative excess demand, the strength of the interaction $J$ and the switch reactivity of agents decisions may determine a loss of stability through two possible bifurcation paths: first, a supercritical Neimark-Hopf bifurcation with periodic or a quasi-periodic motion and
second a supercritical pitchfork bifurcation with the magnetization phenomenon, typical of social interaction models. Furthermore, the model exhibits strong path dependence. Nevertheless, the model in Bischi et al. is unable to explain a period of financial distress (PFD) preceding a crash unless we introduce a financial constraint in agents’ decisions.

Simulations show that the PFD can arise from agents’ wealth distribution dynamics. It is the time necessary, after the bubble’s peak, for a sufficient mass of agents needing to sell because of financial constraints. An increase in the switching strategy velocity (the intensity of choice in the social interaction literature) increases the PFD’s length and decreases the bubble’s amplitude. Finally, an increase of the strength of the interaction increases the amplitude of the bubble. Thus we can model for the first time one of the most prevalent phenomena in actually existing bubbles.

References


Farrell, Michael J., “Profitable Speculation,” Economica, XXXIII (1966), 283-293.


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1 The earlier editions were in 1978, 1989, and 1996. All of them had this appendix, to which Kindleberger kept adding more bubbles with each edition. In the first edition he listed 37, with 30 of them having clearly distinct dates for the peak and the subsequent crash.

2 The term originally comes from corporate finance where a firm is said to be in financial distress when it is unable to make payments to cover its liabilities in the foreseeable future. In this paper investors in an asset market face liquidity constraints that become more severe as prices decline.

3 One can argue that the 1997-98 episodes were really two bubbles and crashes, which would make the numbers 36 out of 47. Adding the US silver and NASDAQ bubbles would make this 36 out of 49, still a solid majority. Even the NASDAQ bubble arguably exhibited a period of financial distress as the day of its most rapid decline occurred nearly a month after its peak in March, 2000.

4 An exception to this general pattern appears to have been the real estate bubble in Japan, which rose sharply in the late 1980s, reached a peak in 1991, and has been more gradually declining ever since [Land Information Division, 2002, Chart 1-1-1].

5 We use the framework of Bischi et al. that considers the benefit function as a potential of a Gibbs distribution.

6 The simulations were performed using Matlab version 6.5.