Herd Behavior and Non-Fundamental Asset Price Fluctuations in Financial Markets

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Aim of the work

Old question: financial time series are explained, essentially, by the dynamics of fundamentals? (Chiarella, 1992; Lux, 1998; Brock and Hommes, 1997b, Chiarella et al., 2001, Chiarella and He 2002).

In this paper, we relax the distinction between fundamentalists and non fundamentalists to put the focus on the role of interaction among individuals during continuous trading. The aim is to investigate the dynamics emerging in a stock market, when in the investment behavior of its participants we allow for a tendency to mimic the actions of other investors, i.e., to engage in herd behavior.

The literature showed that herd behaviors are empirically relevant and may be rational (Banerjee, 1992, Bikhchandani et al., 1992 and Welch, 1992).
The model

\( (i) \) There exist 2 assets: a risk free asset with a constant real return on investment \( r \) and a risky asset with price \( P(t) \) that pays a dividend, say every year, supposed to be an IID stochastic process with mean \( d \).

\( (ii) \) The number of agents who decide to trade the risky asset in period \( t \), \( N(t) \), is a stationary process – with mean \( N \) – independent from agents’ decisions.
(iii) Agents observe past prices, the relative excess demand, \( w(t) = N(t)^{-1} \sum w_i(t) \), the real interest rate, \( r \), and have rational expectations about the dividend (their expected value is equal to \( d \), the mean of the process). This means that the information set of the agent is the union of his/her private characteristics, say the set \( \Omega_i(t) \), and the public information set \( \Omega(t) = \{r, d, w(t-1), w(t-2), \ldots, P(t-1), P(t-2), \ldots\} \).

(iv) In order to take their buy/sell decision, the agents have to evaluate an expected benefit function \( V_i(t) \), that will depend on their prior believes on the price that will prevail in the market. We assume that the agents engage in rational herd behavior, i.e., they expect that \( V_i(t) \) will be positively related with the other agents’ buy/sell decisions.
\( (v) \) Price dynamics is assumed to follow the difference equation

\[
p(t + 1) - p(t) = f(w(t)) \tag{1}
\]

where \( p(t) \) is the logarithm of \( P(t) \), and \( f(w(t)) \) is a deterministic term, that measures the influence of excess demand on current price variations, with properties: \( f(0) = 0 \), \( f'(w(t)) > 0 \).

\( (vi) \) Agents have homogeneous expectations on the relative excess demand at period \( t \), say \( w(t)^e \). Following Brock-Durlauf (2001), agents’ static expectations with respect to their information set are assumed; \( i.e., E[w(t) | \Omega(t)] = w(t - 1) \).
Law of large numbers

Since the $N(t)$ random variables, $w_i(t)$, are independent, conditionally on agents’ expectation, $w(t)^e$, the average choice, will converge to the expected value due to the law of large numbers,

$$\mathbb{E}(w_i(t) \mid w(t)^e). \quad (2)$$

To compute such expectation we need to define the

Benefit function

$$V_i(t + 1, w_i(t)) = (\bar{p}(t) - p(t))w_i(t) + J(t)w_i(t)w(t)^e + \varepsilon_i(t + 1, w_i(t))$$
Implications

It implies that the utility, or benefit function, is affected by three additive components. The first component gives the private benefit in choosing strategy $w_i(t)$. The second one is an interaction term measuring the benefit of that choice in a situation where the expected average choice is $w(t)^e$. Finally, the last term introduces, stochastically and from the point of view of the modeler, idiosyncratic factors affecting agents decisions.

$$J(t) = J(p(t))$$ is a decreasing function of $|\bar{p}(t) - p(t)|$ with $J(\bar{p}(t)) < \infty$
Equilibrium in the expectations

The expected aggregate mean value of the market converges to the quantity

\[ w(t) = \tanh\{\beta(\bar{p}(t) - p(t) + J(t)w(t^e))\} \]  (3)

This equation defines a relation between the actual aggregate excess demand and the value expected by the agents. The expectation equilibria, then, are the points \( w^* \) satisfying the equation

\[ w^* = \tanh\{\beta(\bar{p}(t) - p(t) + J(t)w^*)\} \]

This equilibrium in the expectations, however, cannot be a consistent equilibrium for the market.
Adaptive adjustment mechanism for the priors on the expected price\(^\dagger\):

\[ \bar{p}(t+1) = \bar{p}(t) - \rho (\bar{p}(t) - p(t)) \]  \(4\)

Putting together equations (1), (3) and (4), we get the following three-dimensional discrete dynamical system

\[ T_3 : \begin{cases} 
  w(t+1) = \tanh [\beta (\bar{p}(t) - p(t) + w(t)J(t))] \\
  p(t+1) = p(t) + f(w(t)) \\
  \bar{p}(t+1) = \bar{p}(t) - \rho (\bar{p}(t) - p(t)) 
\end{cases} \]  \(5\)

\(^\dagger\)The use of an adaptive rule for expectation formation is made for the sake of simplicity. This kind of rule is consistent with the assumptions in the model since changes in price are determined by changes in expected demand that, in their turn, are determined by previous expected and actual price and so on recursively.
Simple transformation

\[ q(t) = p(t) - \bar{p}(t) \quad , \quad (6) \]

that represents, at each time period \( t \), the difference between the current price and the expected price. Introducing this change of variable into (5) we obtain a topologically conjugate unidirectionally coupled dynamical system in the variables \((w(t), q(t), \bar{p}(t))\) expressed by

\[
T_3 : \begin{cases} 
    w(t+1) = \tanh [\beta (-q(t) + w(t)J(|q(t)|))] \\
    q(t+1) = (1 - \rho) q(t) + f(w(t)) \\
    \bar{p}(t+1) = \bar{p}(t) + \rho q(t)
\end{cases}
\]

\[
(7)
\]

the expected price \( \bar{p}(t) \) is a driven variable and can be easily obtained from the time series \( \{q(\tau), \tau = 0, ..., t-1\} \) by the closed form

\[
\bar{p}(t) = \bar{p}(0) + \rho \sum_{\tau=0}^{t-1} q(\tau) \quad . \quad (8)
\]
Dynamic properties of the unidirectionally coupled system

(a) convergence to the steady state \( E_0 = (0, 0) \), and such convergence may be oscillatory or monotonic;

\[
P(1) = \beta f'(0) + \rho (1 - \beta J(0)) > 0
\]
\[
Det - 1 = \beta (1 - \rho) J(0) + \beta f'(0) - 1 < 0
\]

(b) a situation of bistability, with stable equilibria characterized by positive and negative coordinates respectively, whose basins of attraction are separated by the stable set of the saddle point \( E_0 \);

\[
\beta \left[ J(0) - \frac{1}{\rho} f'(0) \right] > 1 \tag{9}
\]

\( P(1) = 0 \) with \( Det < 1 \) ⇒ magnetization
(c) periodic or quasi-periodic oscillations along closed invariant curves located around the unstable focus $E_0$

$Det = 1$ with $P(1) > 0$
Path dependence

However, even if $\bar{p}(t)$ converges, its limiting value $\bar{p}_L(t)$ is strongly influenced by the transient part of the sequence $\{q(t)\}$, determined by the driving system, before it enters a neighborhood of 0. This implies that $\bar{p}_L(t)$ is highly path dependent, because any change of the initial condition $(w(0), q(0))$ of the driving system causes a change of the asymptotic value $\bar{p}_L(t)$ of the expected price (and consequently of the current price, being $q(t) = p(t) - \bar{p}(t)$ convergent to zero). This means that any exogenous shocks or other historical accidents are “remembered” by the system, i.e. their effects are not cancelled by the endogenous dynamics.
Conclusions

• The system may show convergence to the steady-state.

• Increasing the sensitivity of the risky asset price to the relative excess demand or the interaction between agents may determine a loss of stability: 1) a supercritical Neimark-Hopf bifurcation (periodic or a quasi-periodic motion); 2) a supercritical pitchfork bifurcation: magnetization.

• Path dependence: it seems to be strongly conditioned by the velocity at which agents forecast the expected price compared to the rapidity of asset price movements.