Variety of Behavior of Equity Returns in Financial Markets

Fabrizio Lillo
in collaboration with Rosario Mantegna

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Outline

• Longitudinal vs. cross-sectional quantities
• Variety of a financial portfolio
• Asymmetry of a financial portfolio
• One-factor model
• High frequency analysis
• Conclusions
Longitudinal vs. cross-sectional quantities

Longitudinal quantities
- Mean return of asset i $\mu_i$
- Volatility of asset i $\sigma_i$

Cross-sectional quantities
- Mean return at day t $\mu(t)$
- Variety at day t $\sigma(t)$

Cross-correlation $\rho_{ij}$ between asset i and j
Database and investigated variable

- The investigated market is the New York Stock Exchange during the 12-year period (3032 trading days) January 1987 - December 1998
- The number of stocks \( n \) continuously traded in this time period is 1071.
- The variable investigated is the daily price return, which is defined as

\[
R_i(t + 1) = \frac{Y_i(t + 1) - Y_i(t)}{Y_i(t)}
\]

where \( Y_i(t) \) is the closure price of the i-th stock at day \( t \) \( (t=1,2,..) \)
What is the distribution of returns of N stocks in a given day?

The answer depends on the day!
For each trading day we extract the ensemble return distribution
The ensemble return distribution is non-Gaussian and the central part is conserved for long period of normal activity. The ensemble return distribution changes dramatically during highly volatile periods. For example, during the Black Monday crash we have

Can we better quantify the statistical properties of the ensemble return distribution?
Market average

• The simplest quantity useful to quantify the behavior of the set of stocks returns is the mean

\[ r_m(t) = \frac{1}{n} \sum_{i=1}^{n} R_i(t) \]

• This quantity is just a not weighted index and quantifies the general trend of the market at day \( t \).
We compare our empirical results with numerical and analytical results based on the one-factor model

$$R_i(t) = \alpha_i + \beta_i R_M(t) + \varepsilon_i(t)$$

where $\alpha_i$ and $\beta_i$ are two constant parameters, $\varepsilon_i(t)$ is a zero mean idiosyncratic term characterized by a variance equal to $\sigma^2_{\varepsilon_i}$

- $R_M(t)$ is the market factor. We choose it as the Standard and Poor’s 500 index.
- We estimate the model parameters and generate an artificial market.
Non-Gaussian one-factor model

• We consider two possible choices for the statistics of the idiosyncratic terms

1) Gaussian noise terms
2) Student’s $t$ noise terms $\varepsilon_i = \sigma_{\varepsilon_i} w$

$$P(w) = \frac{C_\kappa}{(1 + w^2/\kappa)^{(\kappa+1)/2}} \approx \frac{1}{w^{\kappa+1}}$$

where $\kappa = 3$
• The one factor model well reproduces the statistical properties of the market average.
Variety

• We introduce the variety as the root mean square of the stocks returns on a given day

\[ V(t) = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (R_i(t) - r_m(t))^2} \]

• The variety quantifies the different behavior observed in a market (or portfolio) on a given day.

• The variety is not the volatility of the index.
 Variety

- The time evolution of the variety presents slow dynamics and several bursts.
- The temporal correlation function of the variety has a slow decay with time.
- There is a correlation between variety and market volatility (Pearson $r = 0.68$; Spearman $r = 0.37$).
The one-factor model actually predicts that the variety increases with \( r_m^2 \) which is a proxy of the market volatility. We prove that

\[
V(t) \equiv v^2(t) + \frac{\Delta \beta^2}{\beta^2} r_m^2(t)
\]

where \( v^2(t) = \frac{1}{N} \sum_{i=1}^{N} [\varepsilon_i(t)]^2 \) is the variety of idiosyncratic part.
The one-factor model assumes that the idiosyncratic terms are independent of the market return.

As a consequence, the variety of idiosyncratic terms \( v(t) \) is constant in time and independent from the mean \( r_m(t) \).

The empirical results show that a significant correlation between \( v(t) \) and \( r_m(t) \) indeed exists. The degree of correlation is stronger for rallies than for crashes.
• The one-factor model is unable to reproduce the statistical properties of the variety of returns of real financial markets.
Asymmetry

• What fraction $f$ of stocks did actually better than the market?

• A balanced market would have $f = 50%$

• If $f > 50\%$, then the majority of the stocks beat the market, but a few ones lagging behind rather badly, and vice versa

• We quantify this information by introducing the asymmetry of the ensemble return distribution, defined as

  \[\text{asymmetry} = \text{mean} - \text{median}\]

Is the asymmetry also correlated with the market average?
There is a strong correlation between asymmetry and market average.

We observe that the ensemble return distribution is

- Negatively skewed → crashes
- Positively skewed → rallies
High frequency analysis

- We perform a high frequency analysis of one of the largest financial crashes of last 20 years, specifically the August 31, 1998 crash at NYSE
- We use the high frequency data recorded in the Trade and Quote database and the investigated period is August 20, 1998 - September 10, 1998
- We consider 3 time horizons $\Delta t = 5$ min, 55 min, 1 trading day
There is a strong correlation between variety and market average also at a intraday time horizon.

This effect is stronger at the opening and at the closure of the trading day.
• The symmetry change in the presence of large movements also at a intraday temporal scale.
  • The asymmetry effect is stronger at shorter time scales.
Conclusions

• We study the cross sectional quantities in a financial market at different time horizons.

• We introduce the variety and we find that it has a long time memory.

• We find that the symmetry properties of the ensemble return distribution change in extreme market days.

• A simple one-factor model is unable to explain the statistical properties of the variety and the change of symmetry in extreme days.