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EMH vs. Phenomenological models

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Summary

- Efficient market hypothesis (EMH)
  - Rational bubbles
  - Limits and alternatives
- Phenomenological models
  - Continuous-time random walks
  - Relevance for *microscopic* market models
- Conclusion and discussion

Efficient market hypothesis (I)

Roughly speaking a market is (*informationally*) efficient if all available information is optimally used used to determine asset prices at each point in time. Assuming *risk neutral* investors:

$$E\left[r_A(t,t+1) \mid I(t) \right] = r_F(t,t+1)$$

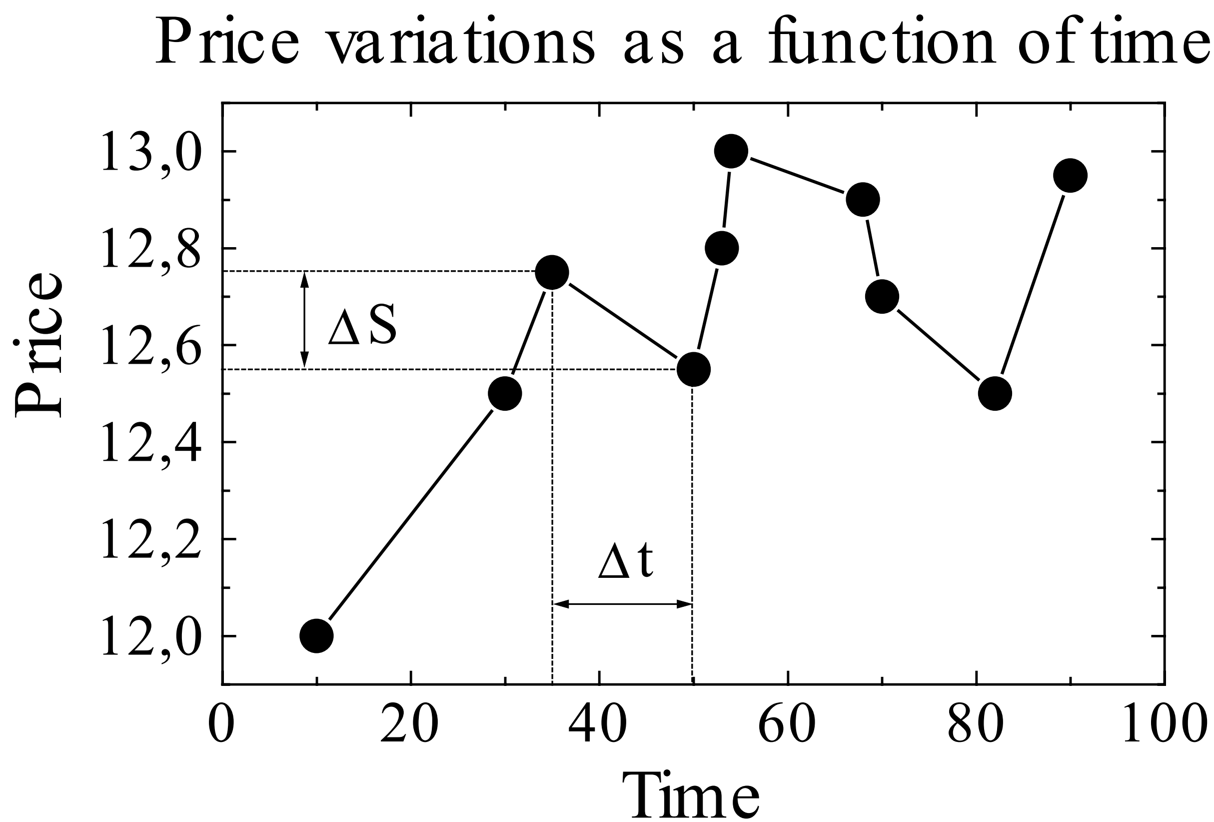
A: risky asset; F: risk free asset; r_A: return rate of the risky asset; r_F: return rate of the risk free asset; I(t): information available to investors at time t. In terms of price $S_A(t)$ and dividends D(t,t+1) payed over the period [t,t+1):

$$\frac{E\left[S_A(t+1) + D(t,t+1) \mid I(t) \right]}{1 + r_F(t,t+1)} = S_A(t)$$

Efficient market hypothesis (II)

- Rational bubbles do exist (bubble: price deviation from the fundamental price). The existence of bubbles is not enough to abandon the EMH. Behavioural assumptions are necessary.
- Noise traders: traders who have wrong pieces of information and believe to have all the available info.
- Zero-intelligence random traders: popular among physicists dealing with microscopic models of market dynamics.
- Our proposal: just consider market phenomenology without behavioural assumptions!

Tick-by-tick price dynamics



Theory (I)
Continuous-time random walk in finance
(basic quantities)

$S(t)$: price of an asset at time ***t***

$x(t) = \log[S(t)]$: log price

$\varphi(\xi, \tau)$: joint probability density of jumps and of waiting times

$$\xi_i = x(t_{i+1}) - x(t_i) \qquad \tau_i = t_{i+1} - t_i$$

$p(x,t)$: probability density function of finding the log price ***x*** at time ***t***

Theory (II): Master equation

$$p(x,t) = \delta(x)\Psi(t) + \int_0^t \int_{-\infty}^{+\infty} \varphi(x-x', t-t')\, p(x',t')\, dt'\, dx'$$

Permanence in x,t

Jump into x,t

$$\lambda(\xi) = \int_0^{\infty} d\tau\, \varphi(\xi,\tau)$$ Marginal jump pdf

$$\psi(\tau) = \int_{-\infty}^{+\infty} d\xi\, \varphi(\xi,\tau)$$ Marginal waiting-time pdf

In case of independence: $\varphi(\xi,\tau) = \lambda(\xi)\psi(\tau)$

$$\Pr(\tau > \bar{\tau}) = \Psi(\bar{\tau}) = 1 - \int_0^{\bar{\tau}} d\tau'\, \psi(\tau')$$ Survival probability

Theory (III): Choice of marginal densities

$$\psi(\tau) = E_{\beta}\left(-\tau^{\beta}\right)$$

$$E_{\beta}(z) = \sum_{k=0}^{\infty} \frac{z^{k}}{\Gamma(\beta k + 1)}$$

$$0 \le \beta \le 1$$

Mittag-Leffler function

$$\lambda(\xi) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \exp(-ik\xi) \exp\left(-|k|^{\alpha}\right) dk$$

$$0 \le \alpha \le 2$$

Lévy function

Results (I)

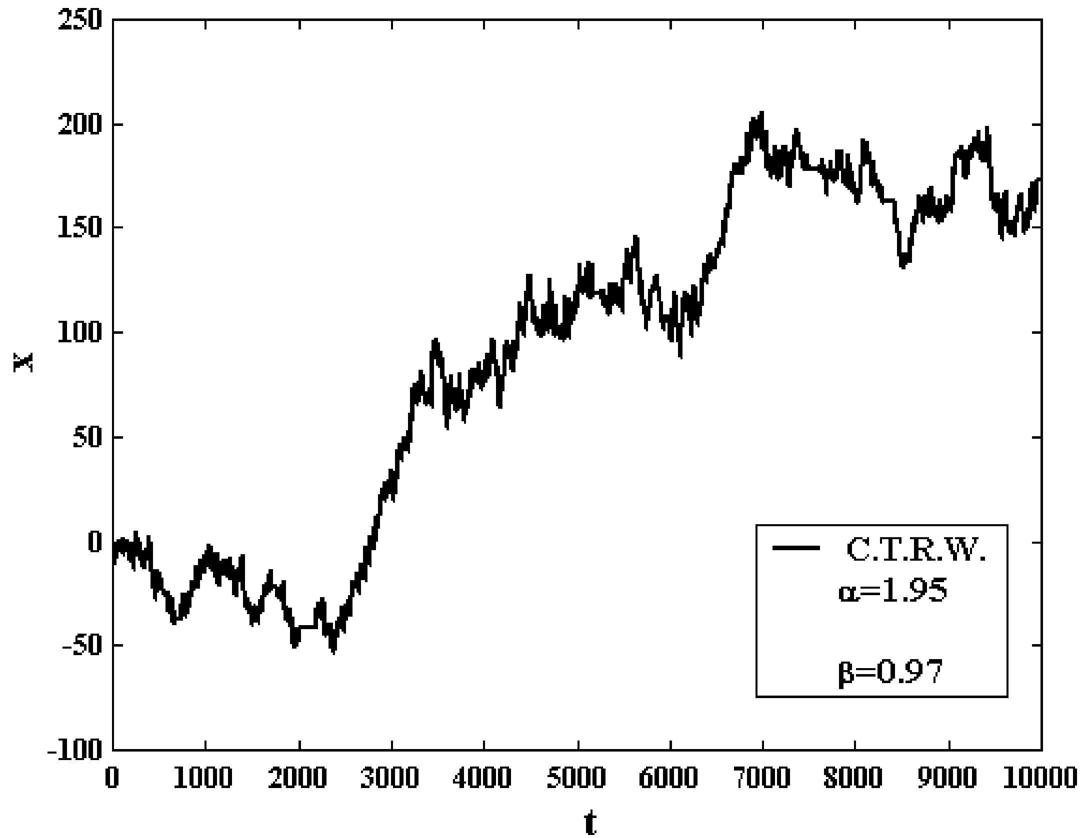


Fig. 1: Simulated log-price as a function of time. Both quantities are plotted in arbitrary units. This simulation includes nearly 8300 log-prices. It takes a few seconds to run on an old Pentium II processor at 349 MHz.

Results (II)

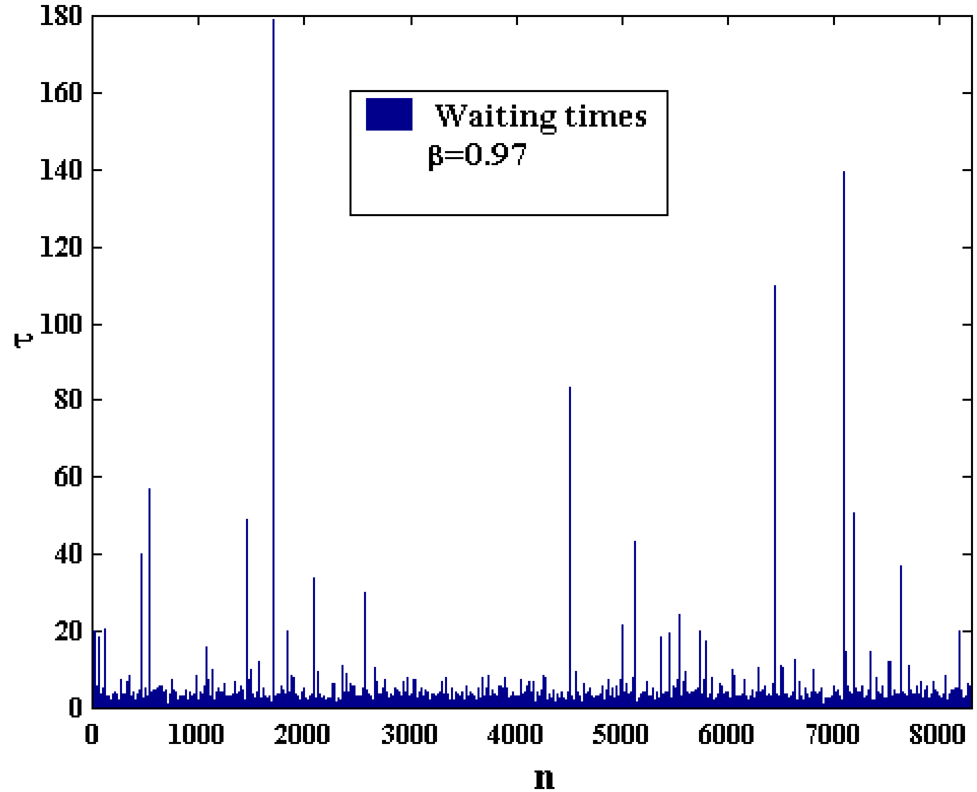


Fig 2: Waiting times as a function of the simulation index.
The series has been produced by means of the rejection procedure, by comparing [0-1]-uniformly distributed deviates to a probability density of the Mittag-Leffler form with $\beta = 0.97$.

Results (III)

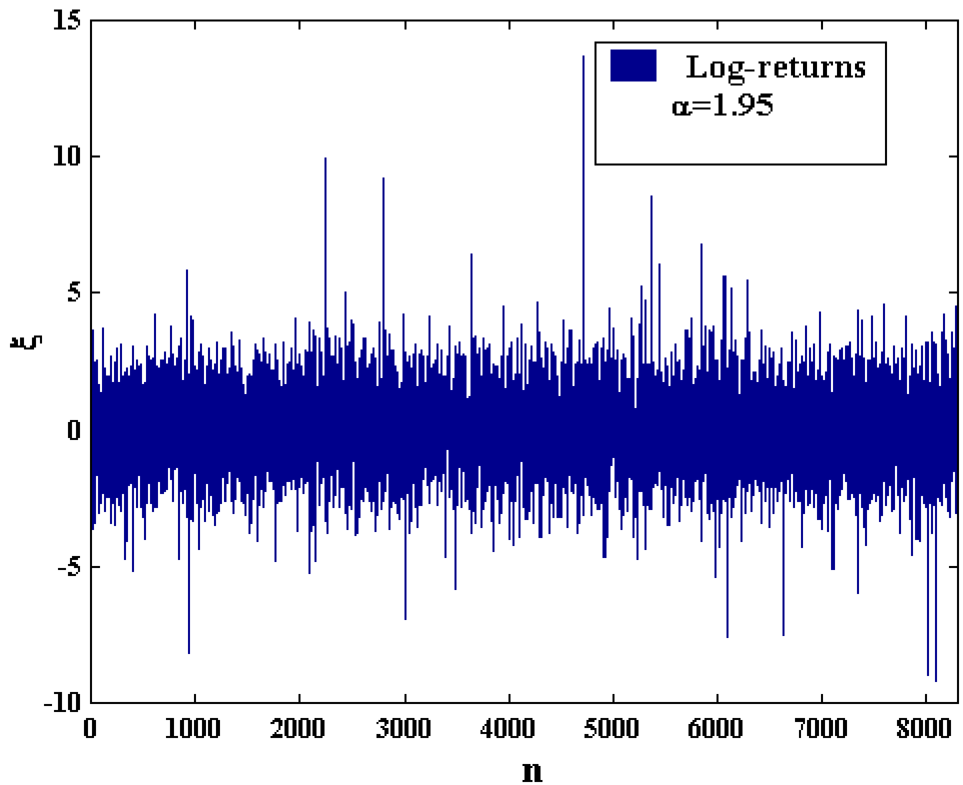


Fig. 3: Log-returns as a function of the simulation index.
The series has been produced by means of the rejection procedure, by comparing [0-1]-uniformly distributed deviates with a Lévy probability density of index α = 1.95.

Results (IV)

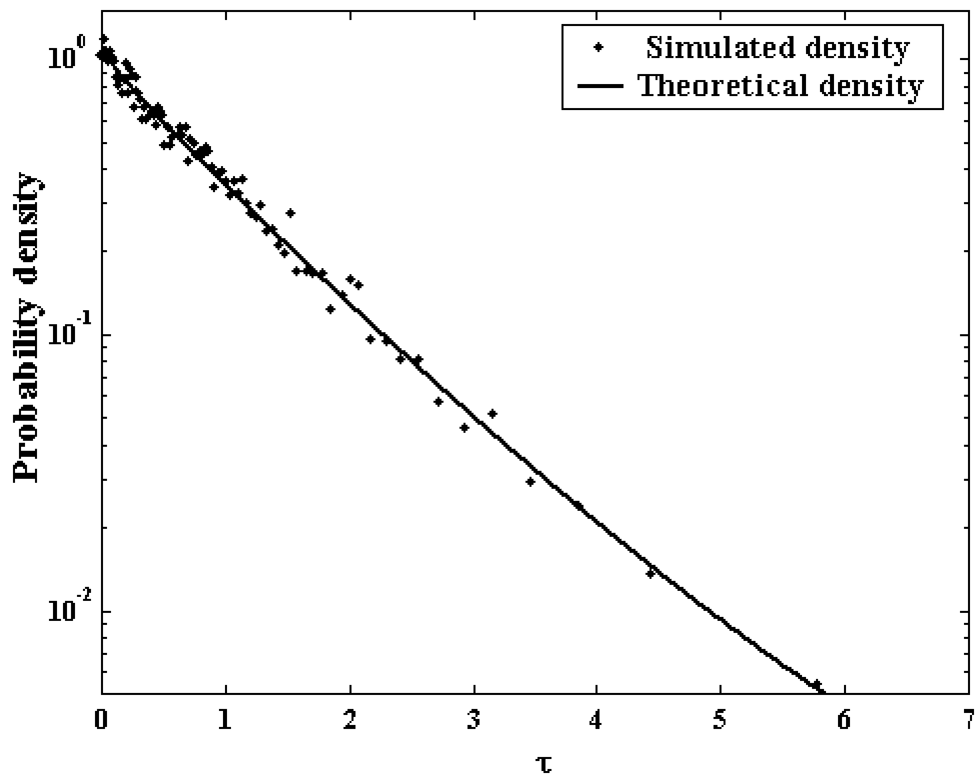
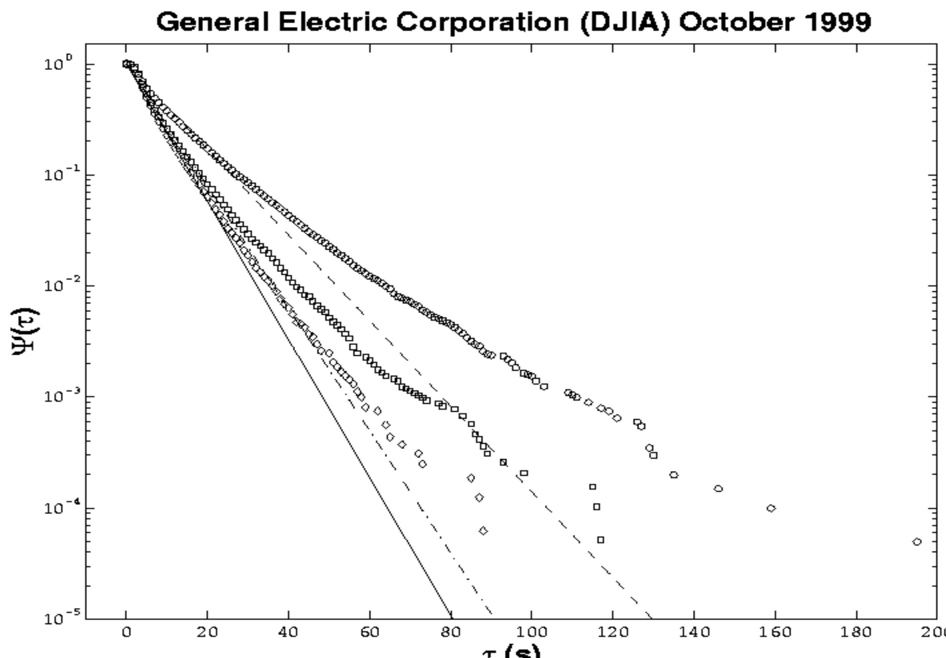


Fig. 4: Waiting times: frequency histogram of simulated data compared with theoretical values.

This plot is a self-consistency check for the simulation procedure.

Empirical results on the waiting-time survival function and their relevance for market models (Anderson-Darling test) (I)

Interval 1 (9-11): 16063 data; τ_0 = 7 s
Interval 2 (11-14): 20214 data; τ_0 = 11.3 s
Interval 3 (14-17): 19372 data; τ_0 =7.9 s



$$A^2 = \left(- \sum_{i=1}^{n} \frac{(2i-1)}{n} \left[\ln \Psi(\tau_{n+1-i}) + \ln(1-\Psi(\tau_i)) \right] - n \right) \times (1 + (0.6/n))$$

where $\tau_1 \leq \tau_2 \leq \ldots \leq \tau_n$

A_1^2= 352; A_2^2= 285; A_3^2= 446 >> 1.957 (1% significance)

Empirical results on the waiting-time survival function and their relevance for market models (Anderson-Darling test) (II)

- Non-exponential waiting-time survival function now observed by many groups in many different markets (Mainardi et al. (LIFFE) Sabatelli et al. (Irish market and), K. Kim & S.-M. Yoon (Korean Future Exchange)).

- Why should we bother? This has to do both with the market price formation mechanism and with the bid-ask process.
  If the bid-ask process is modelled by means of a Poisson distribution (exponential survival function), its random thinning should yield another Poisson distribution. This is not the case!

- A clear discussion can only be found in a recent contribution by the Genoa GASM group (http://cinef.dibe.unige.it/wehia2003/DoubleAuction.pdf).

- Question sent to the Santa Fe group: only interlocutory answer received so far.

Results (V)

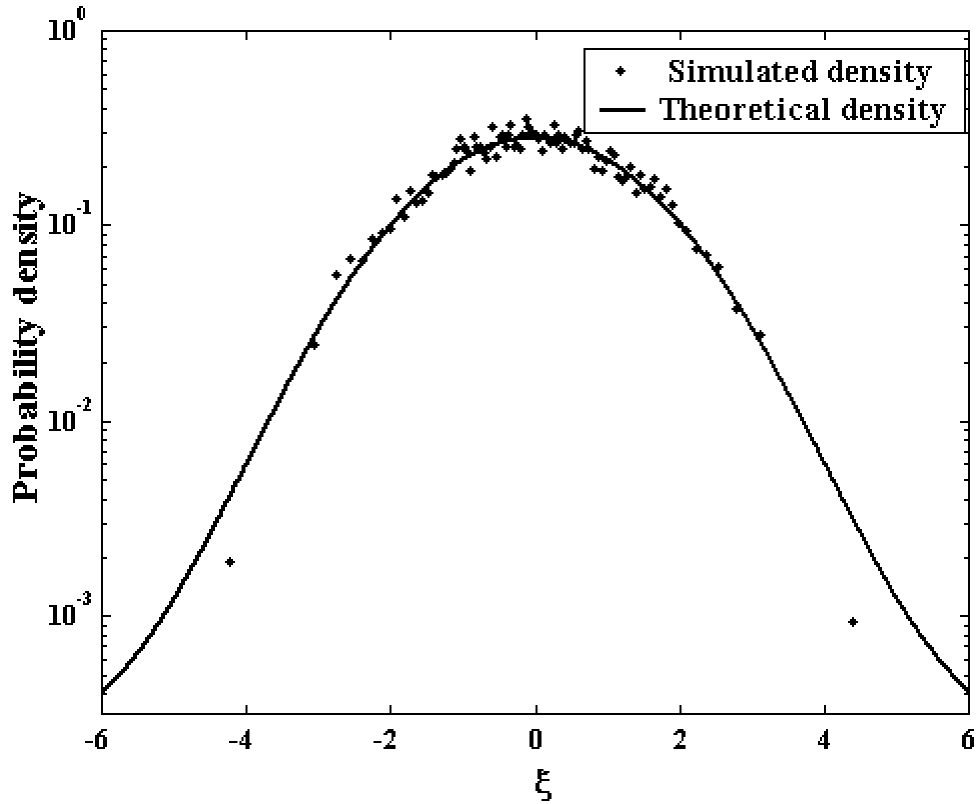


Fig 5: Log-returns: histogram of simulated data compared with theoretical values.
This plot is a self-consistency check for the simulation procedure.

Results (VI)

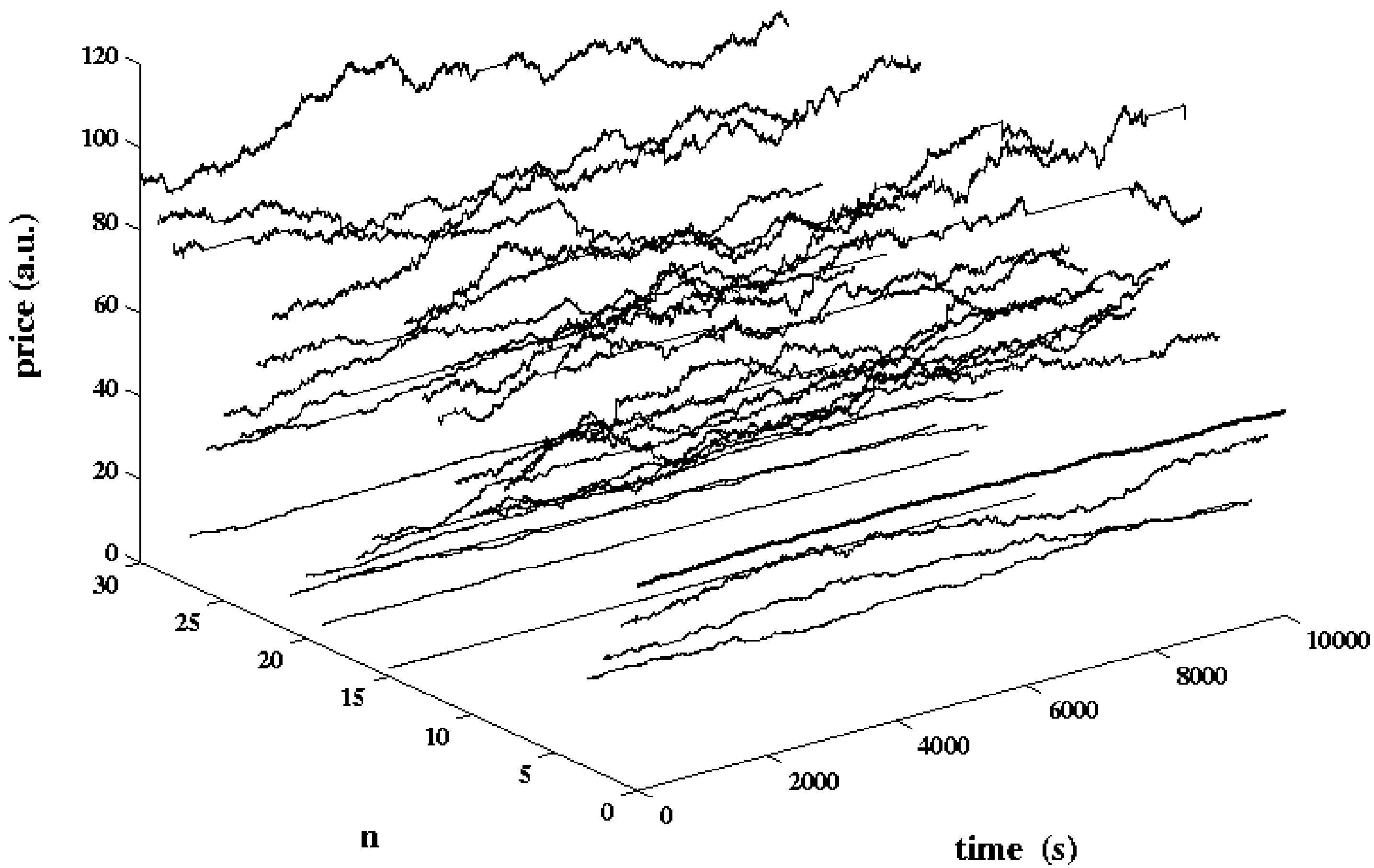


Fig 6: A synthetic market of 30 stocks.

Results (VII)

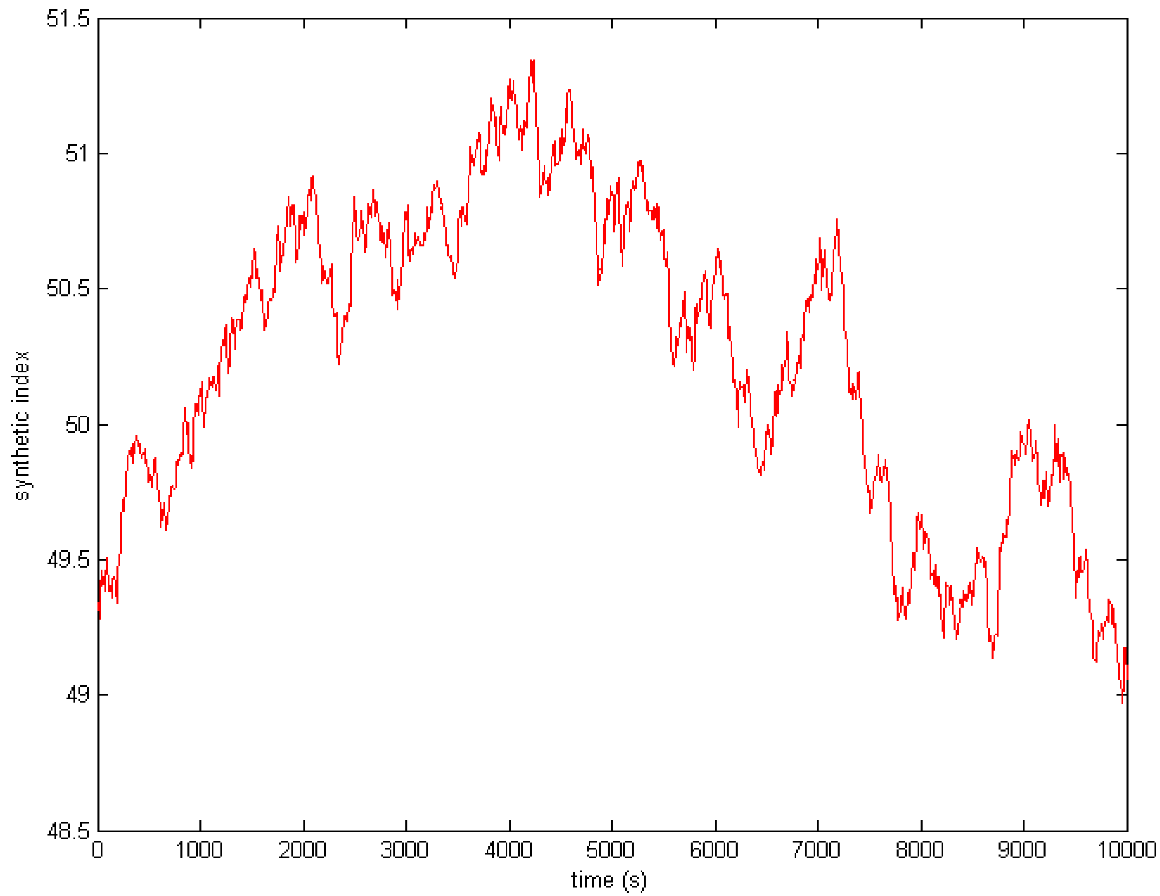


Fig 7: Synthetic market index. The index is the average of the market prices sampled every ten seconds

Conclusions

- CTRWs can be used as phenomenological models for high-frequency market dynamics.
- A synthetic market has been simulated for a specific choice of waiting-time and jump p.d.fs.
- Such simulations can be of help in various applications.

Basic references

[1] E.W. Montroll and G.H. Weiss, *Random walks on lattices II*, J. Math. Phys. **6**, 167-181, 1965.

[2] E. Scalas, R. Gorenflo and F. Mainardi, *Fractional calculus and continuous-time finance*, Physica A, **284**, 376-384, 2000.

[3] F. Mainardi, M. Raberto, R. Gorenflo and E. Scalas*, Fractional calculus and continuous-time finance II: the waiting-time distribution*, Physica A, **287**, 468-481, 2000.

[4] M. Raberto, E. Scalas and F. Mainardi*, Waiting-times and returns in high-frequency financial data: an empirical study*, Physica A, **314**, 749-755, 2002.

Other references

[5] M. Porto and H. E. Roman, *Autoregressive processes with exponentially decaying probability distribution functions: Applications to daily variations of a stock market index*, Phys. Rev. E. **65**, 046149, 2002.
[6] C. Dose, M. Porto, and H. E. Roman, *Autoregressive processes with anomalous scaling behavior: Applications to high-frequency variations of a stock market index*, Phys. Rev. E, in press.
[7] L. Sabatelli, S. Keating, J. Dudley and P. Richmond, *Waiting time distribution in financial markets*, Eur. Phys. J. B **27**, 273-275, 2002.
[8] J. Masoliver, M. Montero, and G.H. Weiss, *Continuous-time random-walk model for financial distributions*, Phys. Rev. E **67**, 021112/1-9, 2003.
[9] W.H. Press, S.A. Teukolsky, W.T. Vetterling, and B.P. Flannery, *Numerical Recipes in C,* p.290, Cambridge University Press, Cambridge UK, 2002.
[10] B. Mandelbrot, *The variation of certain speculative prices*, Journal of Business **36**, 394-419, 1963.
[11] R.N. Mantegna and H.E. Stanley, *Stochastic process with ultraslow convergence to a Gaussian: the truncated Lévy flight*, Phys. Rev. Lett. **73**, 2946-2949, 1994.
[12] I. Koponen, *Analytic approach to the problem of convergence of truncated Lévy flights towards the Gaussian stochastic process*, Phys. Rev. E **52**, 1197-1199, 1995.
[13] M.G. Mittag-Leffler, *Sur la nouvelle fonction* $E\alpha\ (\xi)$, Comptes Rendus Acad. Sci. Paris, **137**, 554–558, 1903.
[14] M.G. Mittag-Leffler, *Sur la representation analytique d'une branche uniforme d'une fonction monogene*. Acta Math. **29**, 101-181, 1905.
[15] F. Mainardi and R. Gorenflo, *On Mittag-Leffler-type functions in fractional evolution processes*, J. Computational and Appl. Mathematics, **118**, 283-299, 2000.
[16] A. Erdélyi, W. Magnus, F. Oberhettinger, and F.G. Tricomi, *Higher Transcendental Functions,* vol. III, Ch. 18, Krieger, Malabar FL, USA, 1981.