Observatory of
Complex Systems
Palermo University
INFM, Palermo Unit

SANTA FE
INSTITUTE

Indagini Empiriche di Dati di Alta Frequenza in Finanza

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Outline

I will present three empirical results recently obtained in the analysis of high frequency data in finance

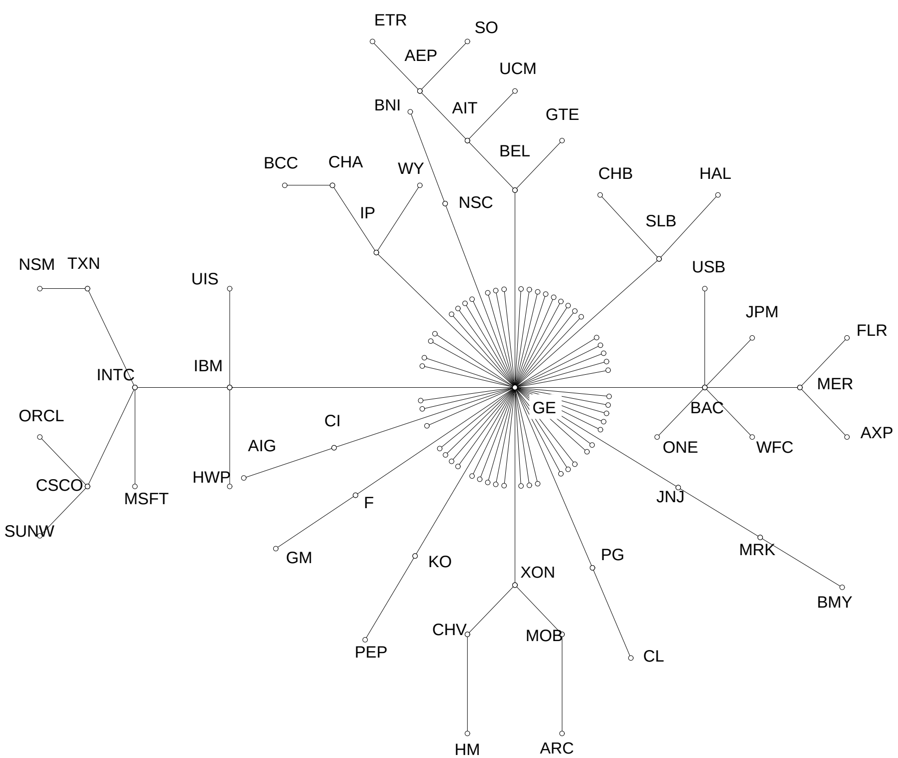
- High frequency cross correlation (stationary property)
- Omori law in an aftercrash period (non-stationary property)
- Universal price impact function (microstructure)

Cross-correlation at different time horizons

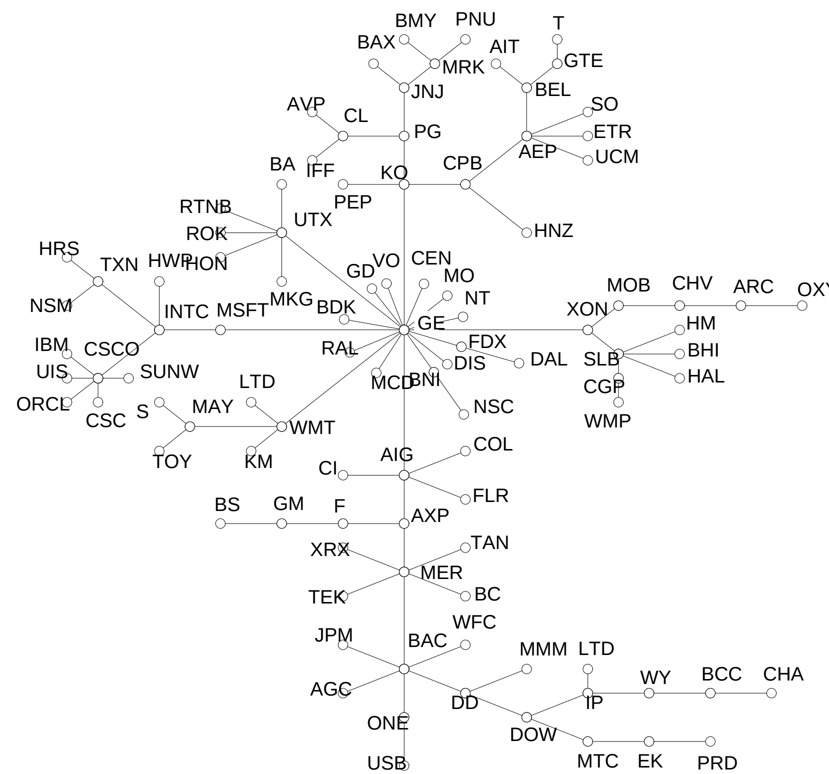
- We use correlation based techniques to uncover the cluster structure of a financial portfolio

How does the correlation structure change with time horizon used to compute correlation coefficients?

Δt=20 minutes

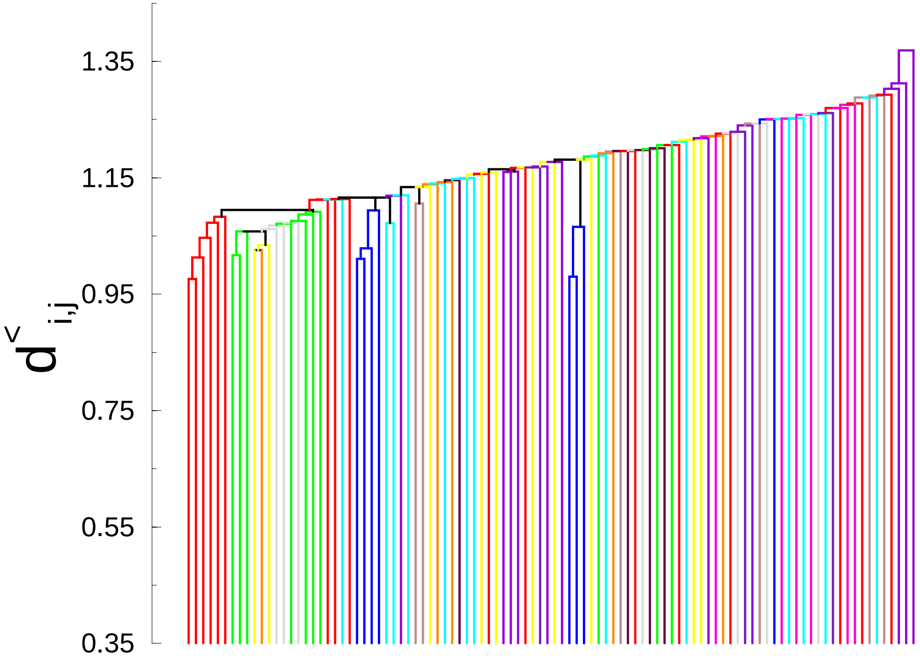


Δt=1 trading day

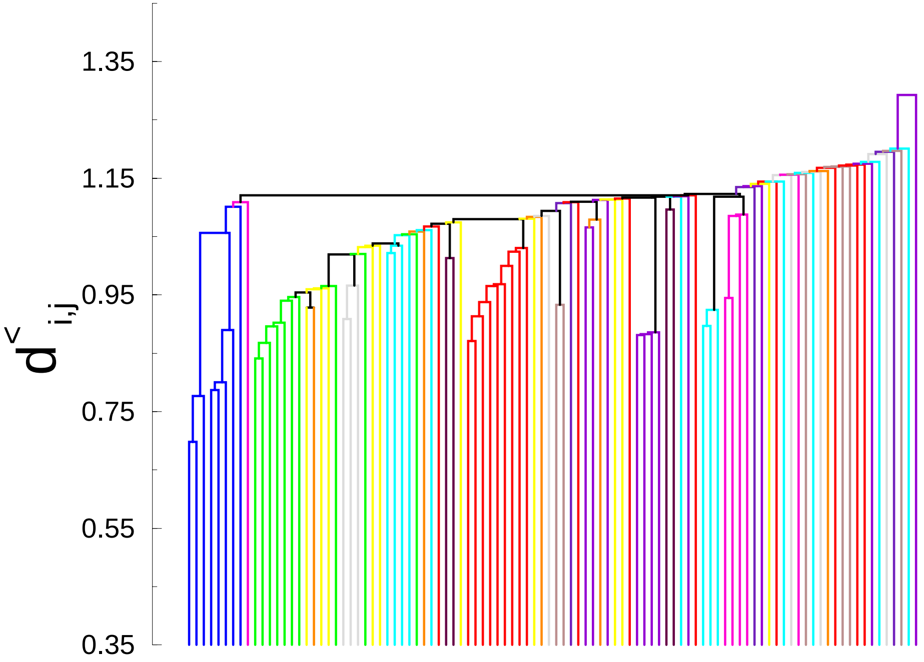


Hierarchical trees

Δt=20 minutes



Δt=1 trading day

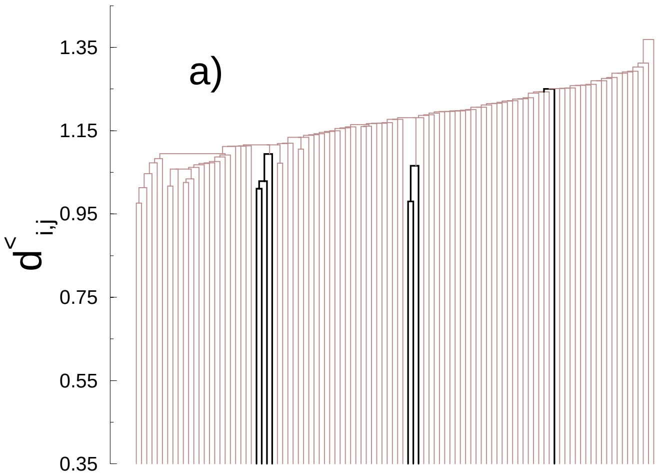


Bonanno, Lillo, Mantegna, *Quantitative Finance* 2001

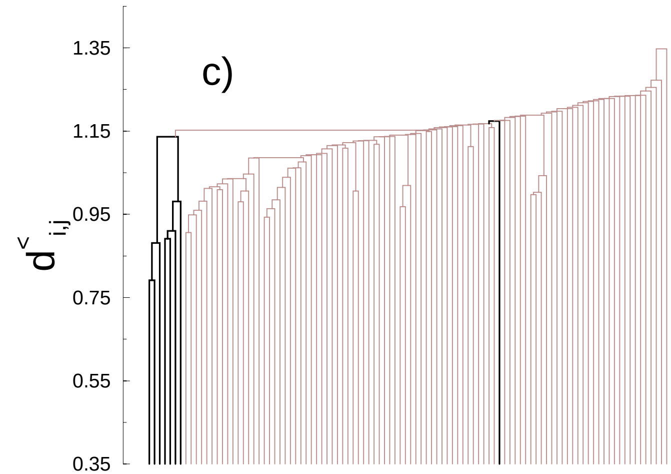
What is the dynamics of clustering ?

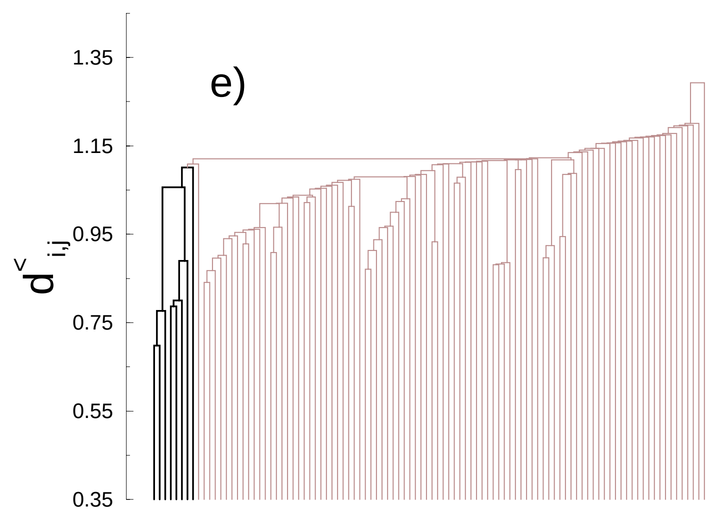
a case study: the energy sector

Δt=20 minutes



Δt= 78 minutes



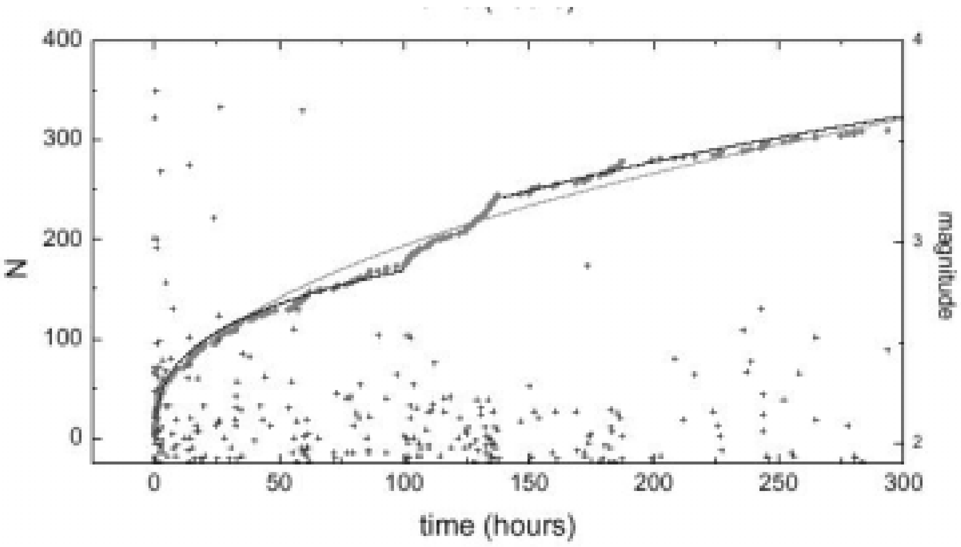


Δt=1 trading day

Characterization and modeling of aftercrash period

The Omori law in geophysics is governing the dynamics of the number of aftershocks after a major earthquake.

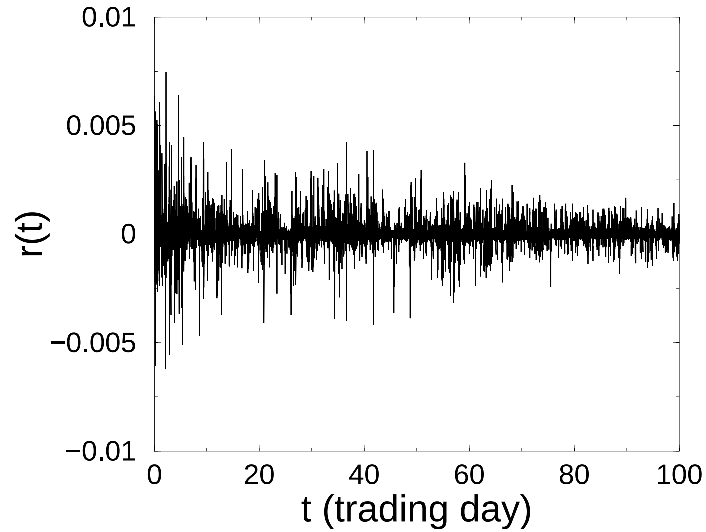
$$n(t) \propto t^{-p}; \qquad N(t) = \int_0^t n(s)ds$$

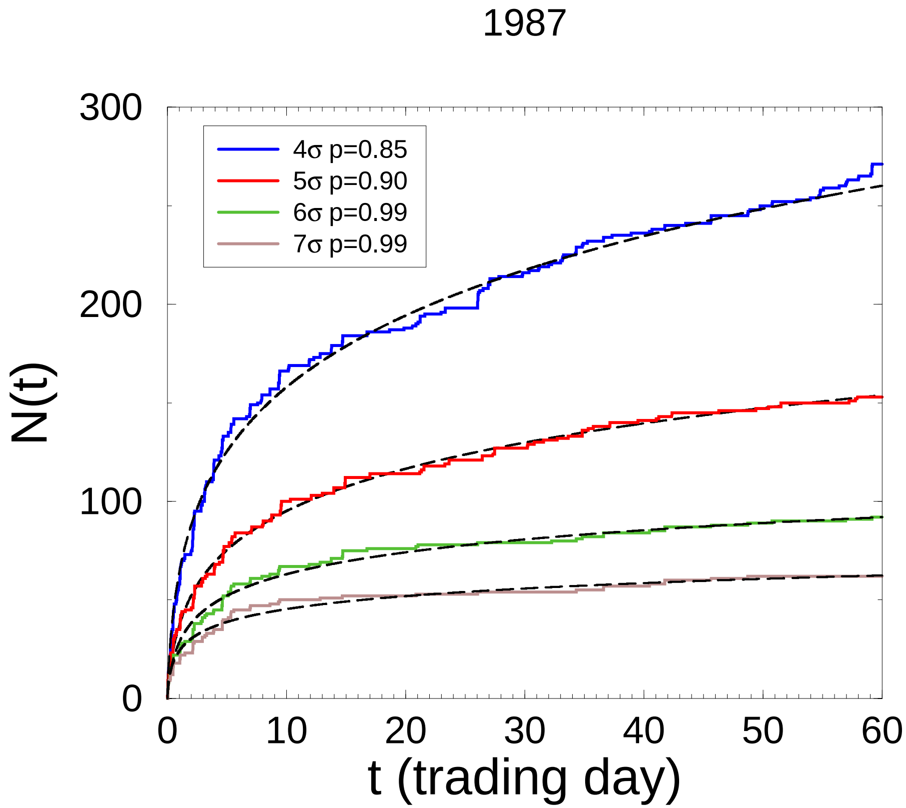


Cumulative number of aftershocks in the earthquake occurring in eastern Pyrenees on February 18, 1996 (from Moreno *et al.*, *J. of Geophys. Res.*, **106 B4**, 6609-6619 (2001)

Characterization and modeling of aftercrash period

One minute return of S&P 500 Index after the **Black Monday** financial crash (19 Oct 1987).





We measure the cumulative number of index returns exceeding a given threshold **nσ** (n=4,5,6,…).

Omori law after the **Black Monday** financial crash.

(Lillo & Mantegna, PRE 2003)

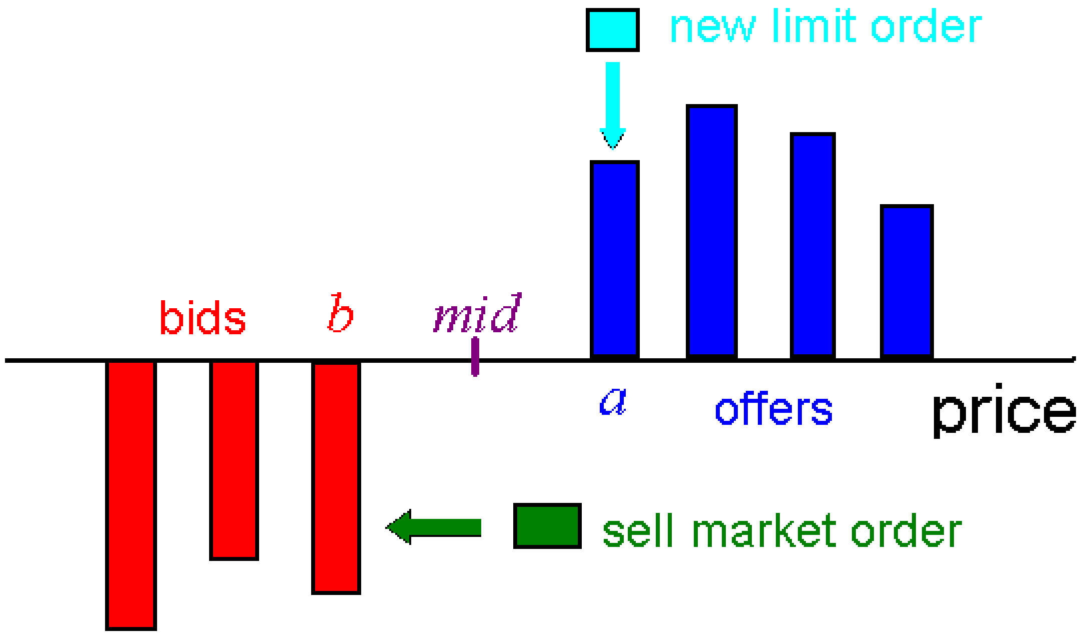
Characterization and modeling of aftercrash period

- We observe the Omori law in other financial crashes
- We show that our empirical results are consistent with a description of return dynamics as the product of an i.i.d. power-law distributed stochastic process times a volatility which is power-law decaying in time.
- We show both analytically and numerically that the Omori law is not explained by GARCH(1,1) type models
- A careful characterization of the Omori law allows to predict the Risk after a big crash (for example, through Value at Risk).
- Omori law could be the macroscopic manifestation of a universal mechanism through which complex systems relax after a big shock. In fact power-law relaxation has been observed in earthquakes, internet traffic, microfracturing phenomena, long range correlated systems, spin glasses

Microstructure analysis: short term price impact in double auction mechanism

Many stock exchanges works through a double auction mechanism

Limit Order Book



How does price react to a single transaction?

In principle the answer depends on many factors, including

- transaction volume
- identity and liquidity of the stock
- properties of the limit order book
- strategies of the agents
- regulations of the market (specialist vs. electronic exchange)

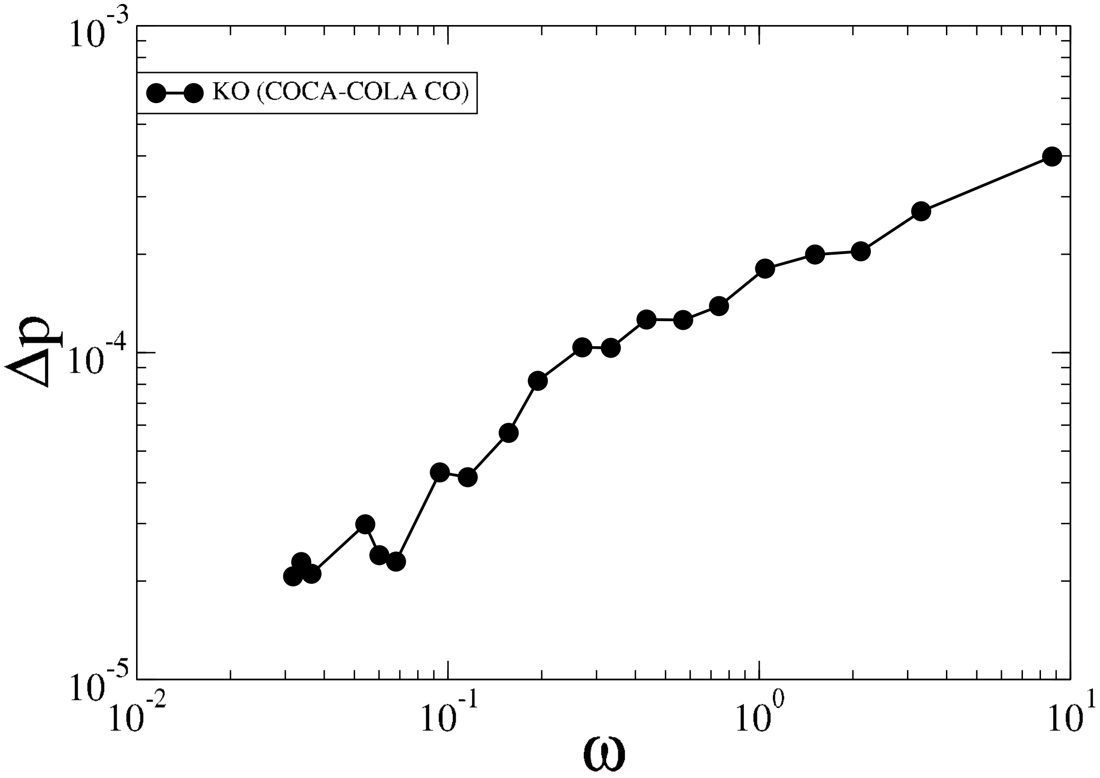
We empirically investigated this problem. Our database consists all the transactions and all the best quotes of the 1000 most capitalized companies traded at NYSE in the period 1995-1998.

The total number of transactions is 10^7 in 1995.

The market capitalization of the investigated companies ranges from 10^7 to 10^{11} \$ in 1995.

Case study: Coca Cola in 1996

- KO is one of the most capitalized companies traded at NYSE: buyer market order



Results

Our single stock analysis shows that the price shift of a single transaction displays two regimes

• For small volumes

$$\Delta p \approx \frac{\sqrt{|\omega|}}{\lambda}$$

where λ is the liquidity parameter

• For large volumes

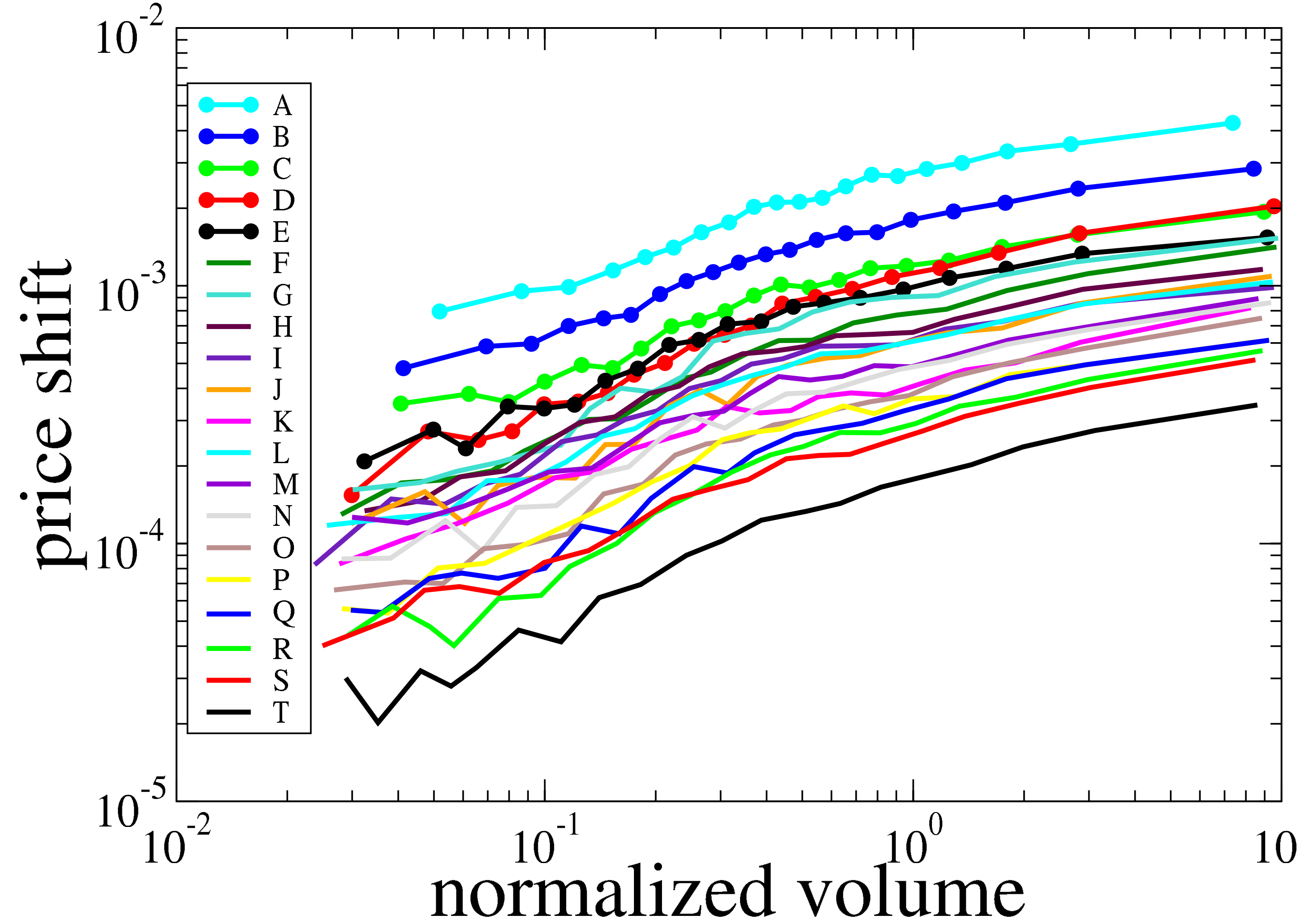
$$\Delta p \approx c|\omega|^{0.2}$$

These results are predicted by a random order placement model (or zero-intelligence model) recently developed (Daniels et al., PRL, 2001).

These findings show that

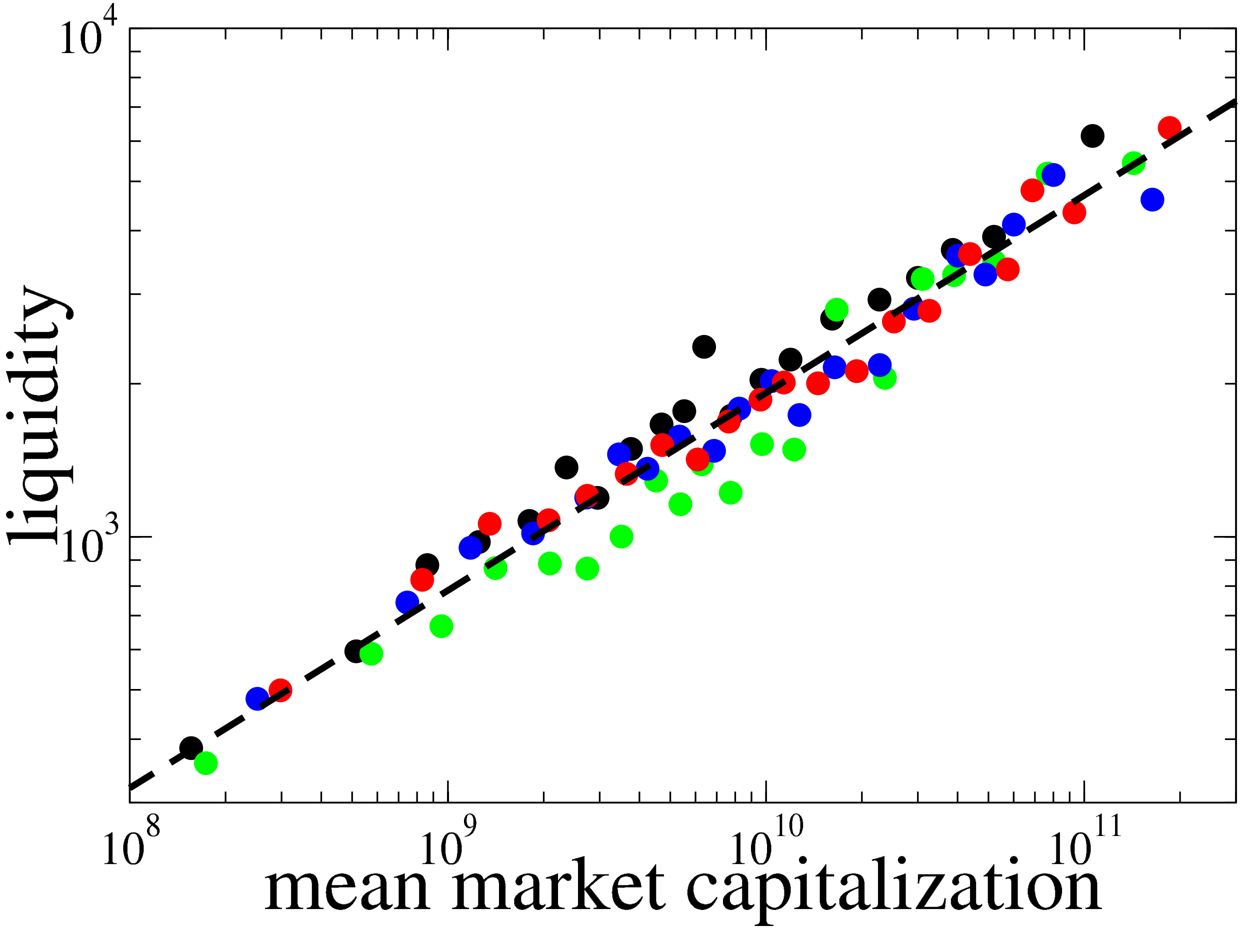
- it is important to model financial institution in detail
- sometimes it can be more useful to model human behavior as random rather than rational

How universal are these findings ?



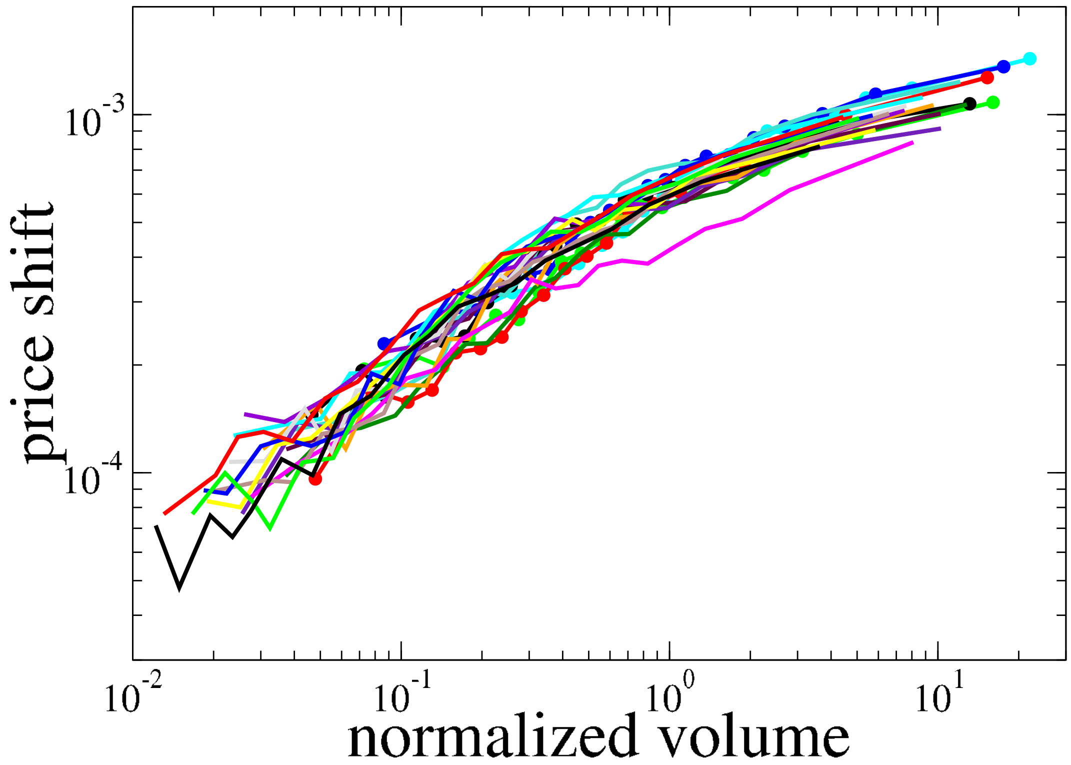
price shift
normalized volume
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-2}
10^{-1}
10^{0}
10^{1}
A
B
C
D
E
F
G
H
I
J
K
L
M
N
O
P
Q
R
S
T

Market capitalization & Liquidity



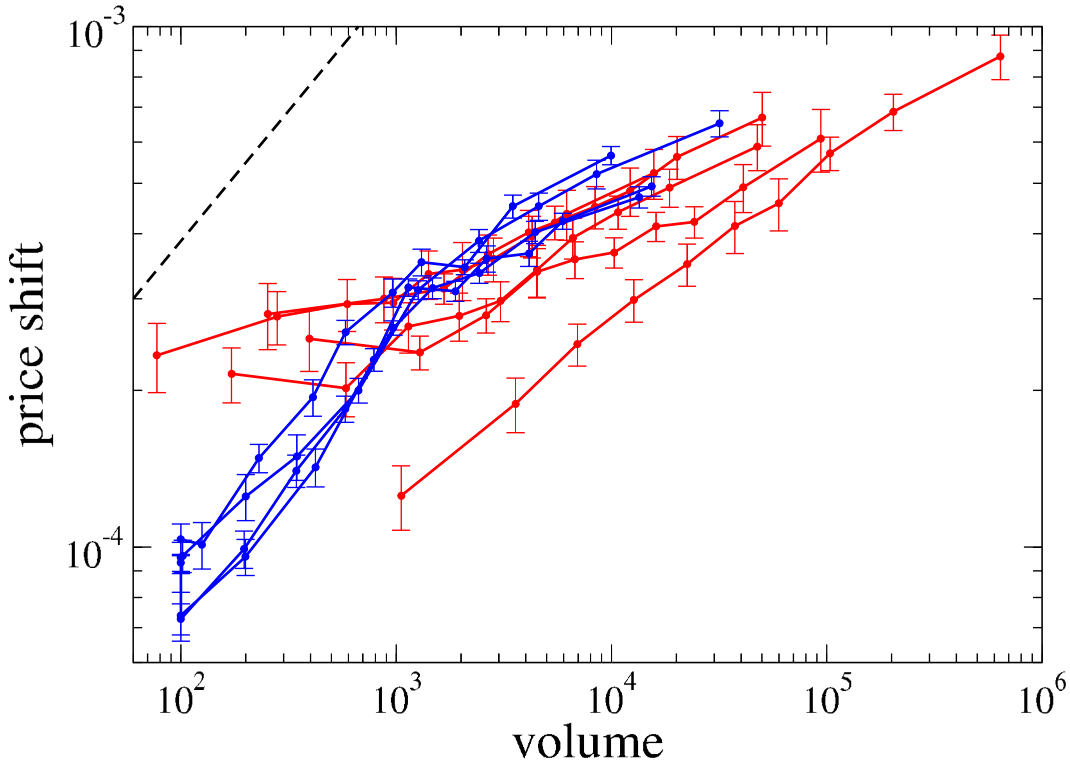
Master curve for price-impact function

(Lillo, Farmer, Mantegna, *Nature*, 2003)



$$\Delta p = C^{-\gamma} f\left(\omega C^{\delta}\right)$$

Does the price impact depend on the market regulations ?



Blue = New York Stock Exchange (specialist market)

Red = London Stock Exchange (electronic market)

Conclusions

- The empirical analysis of high frequency data allows to investigate the dynamics of price formation and information spreading in normal and crash period.
- The microstructure analysis of price impact and of limit order queuing is useful in order to distinguish regularities which are due to agent behavior from those due to the market institution.
- The approach has clear practical applications for example in risk quantification after a large movement of the market or in high frequency trading strategies by using the market impact.