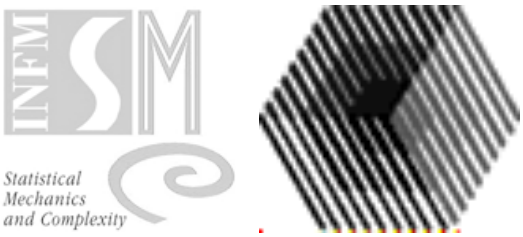


Collective phenomena in systems of competing agents

Andrea De Martino

INFM–SMC and Dipartimento di Fisica
Università di Roma "La Sapienza"
p.le A. Moro 2, 00185 Roma

andrea.demartino@roma1.infn.it

Milano, July 11th 2003

Stylized facts

The statistics of price changes in financial markets exhibits several 'anomalous' (i.e. non Gaussian) fluctuation phenomena

- Power-law tail in the distribution of returns (exponent between 3 and 5)

- Volatility clustering and intermittency

- Long-range volatility correlations
$\langle \sigma(t)\sigma(t+\tau)\rangle \sim \tau^{-\gamma}$, $\gamma \sim 0.1$–0.3

- ...

These are observed in many different markets (stocks, currencies, interest rates,...) and over different periods of time

$\Rightarrow$ Do they originate from a common mechanism?

Agent-based models: minority games

For each trader i:

Map information $\rightarrow$ action

- $\{1, \ldots, P\} \ni \mu \rightarrow a_i^{\mu} \in \{-1, 1\}$ (buy/sell)

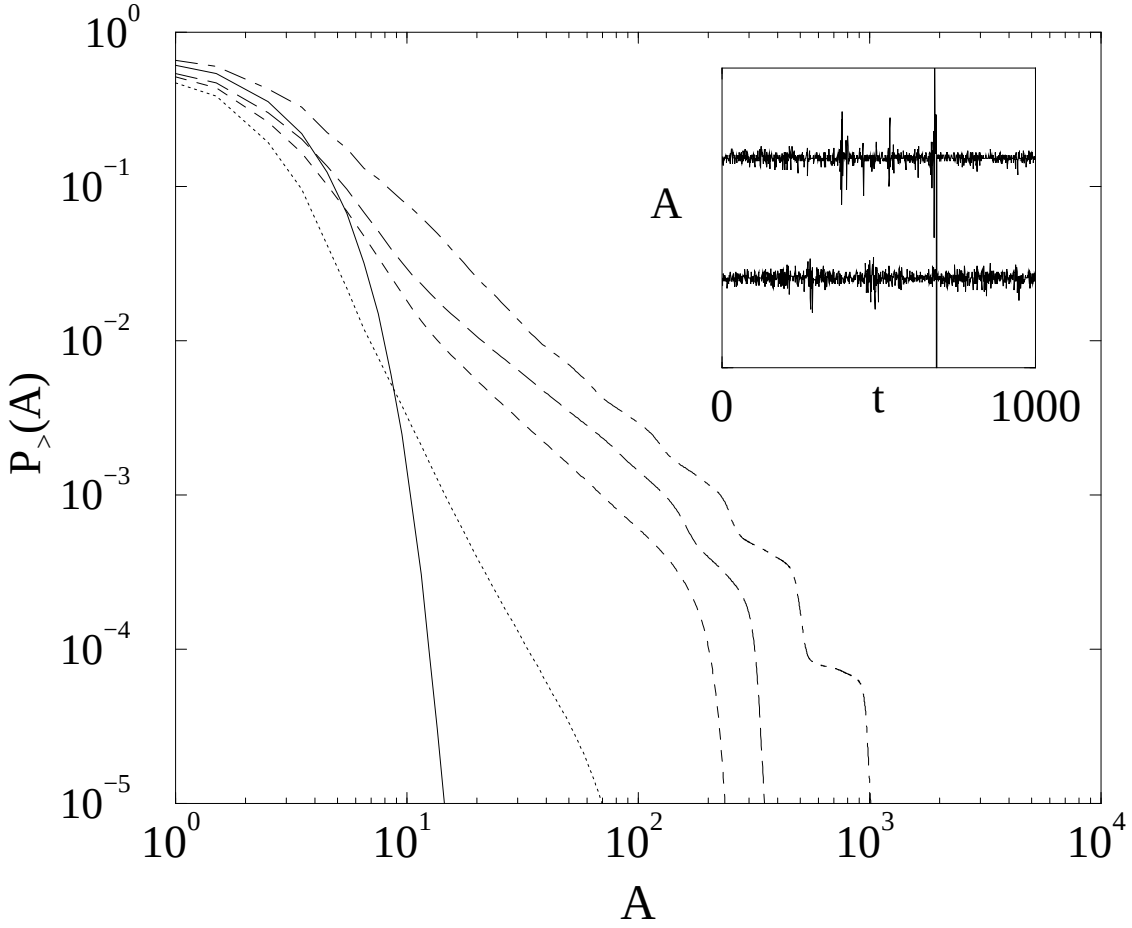
At each time step t, i reacts to the receipt of a (random) μ

- Bid: $b_i(t) = \phi_i(t) a_i^{\mu(t)}$, $\phi_i(t) \in \{0, 1\}$
- Total bid: $A(t) = \sum_i b_i(t)$
- Payoff: $y_i(t+1) = y_i(t) - a_i^{\mu(t)} A(t) - \epsilon_i$
- Decision: $y_i(t+1) > 0 \;\Rightarrow\; \phi_i(t+1) = 1$

ϵ_i describes risk-proneness
$\epsilon_i = -\infty$: agent always trades ('producer')
ϵ_i finite : 'speculator'

(Challet & Marsili 2002)

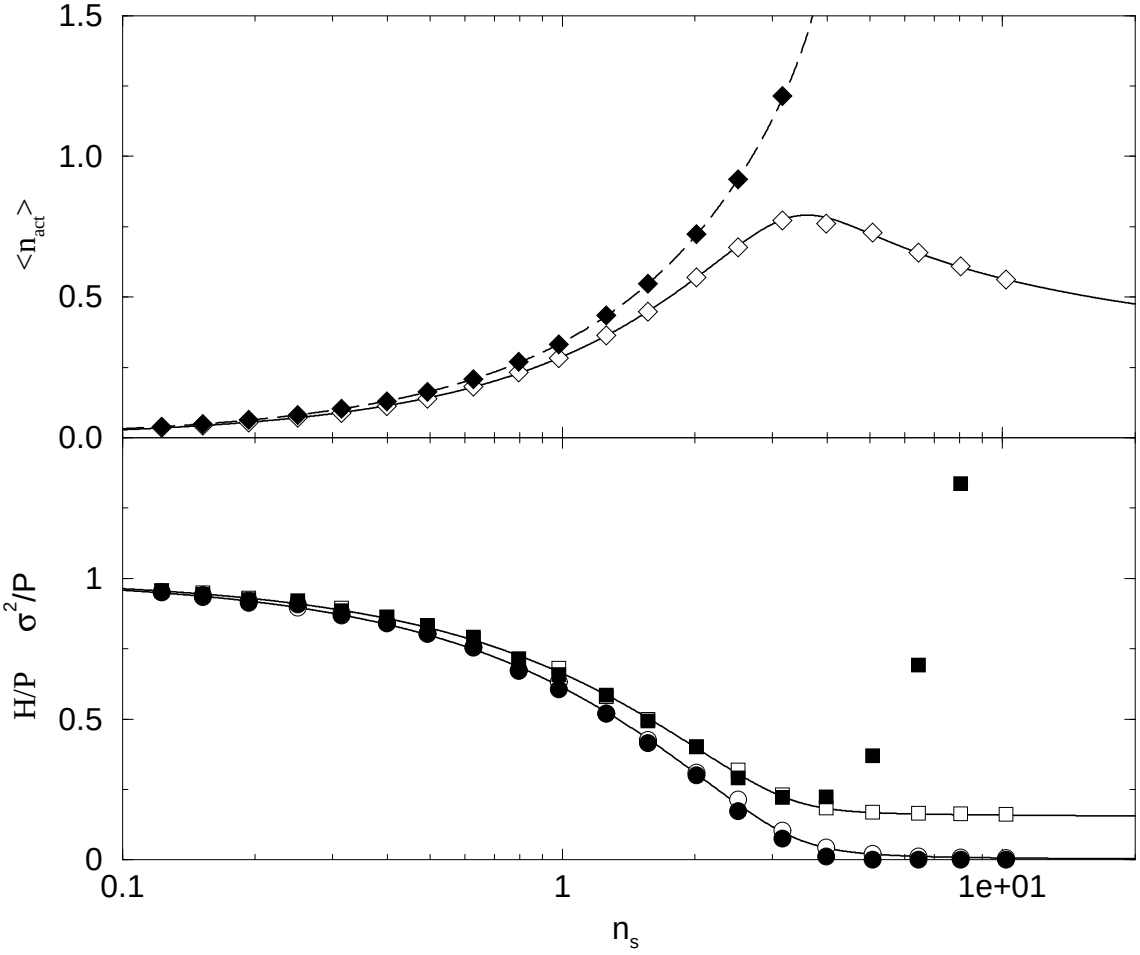
The structure of fluctuations depends on the composition of the market



Increasing the amount of speculators:

$$\text{Prob}\{|A| > x\} \sim x^{-\delta} \qquad \delta \sim 3\text{–}1.4$$

Increasing the amount of speculators at a fixed amount of producers:



Top: fraction of active traders

Bottom: volatility (σ^2) and predictability (H)

$$H \sim \sum_\nu A(t)^2 \delta_{\mu(t),\nu}$$

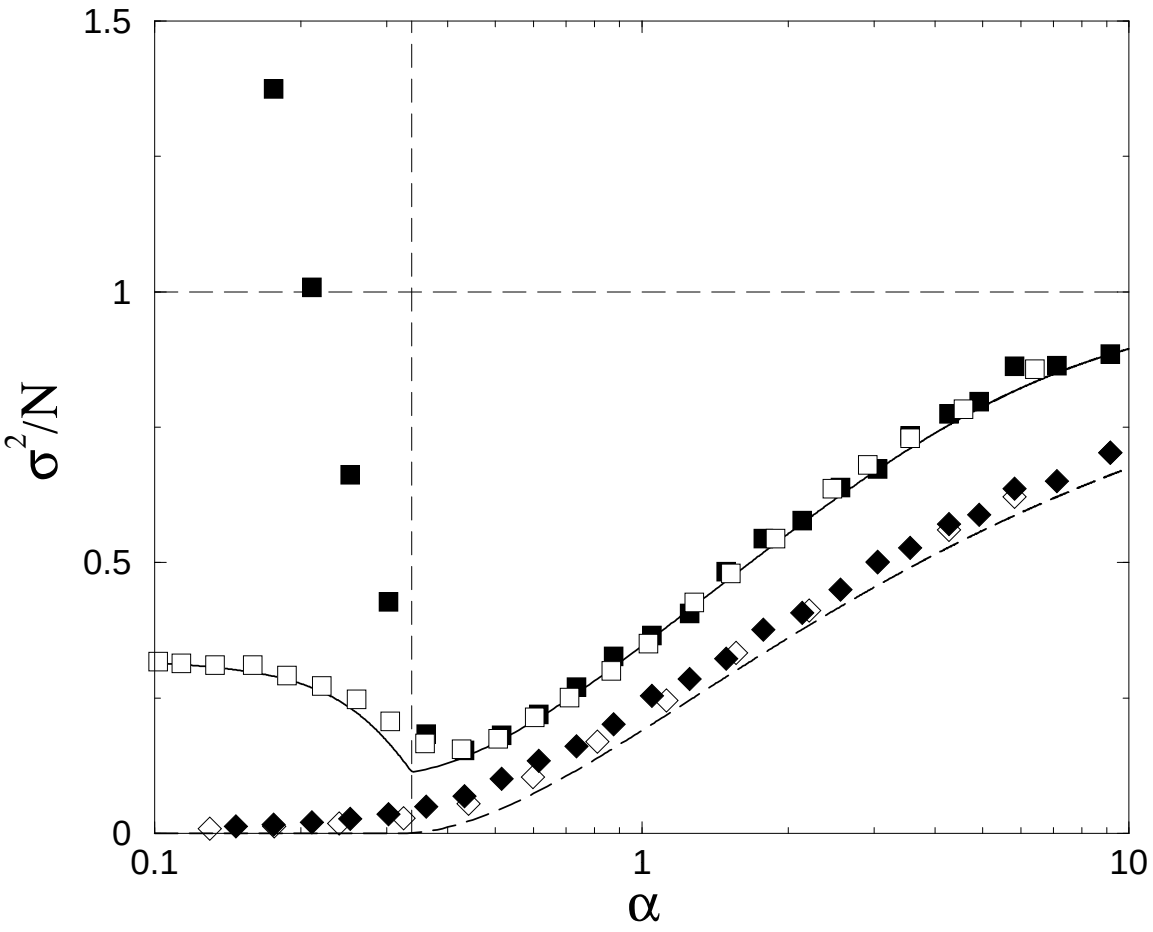
$H > 0$ signals inefficiency

More complicated situations

Agents with more strategies (Challet, Marsili & Zecchina 1999)

- From $\{0,1\} \ni \phi_i(t) \rightarrow s_i(t) \in \{-1,1\}$

If $y_i(t) > 0$ then $s_i(t) = 1$, else $s_i(t) = -1$



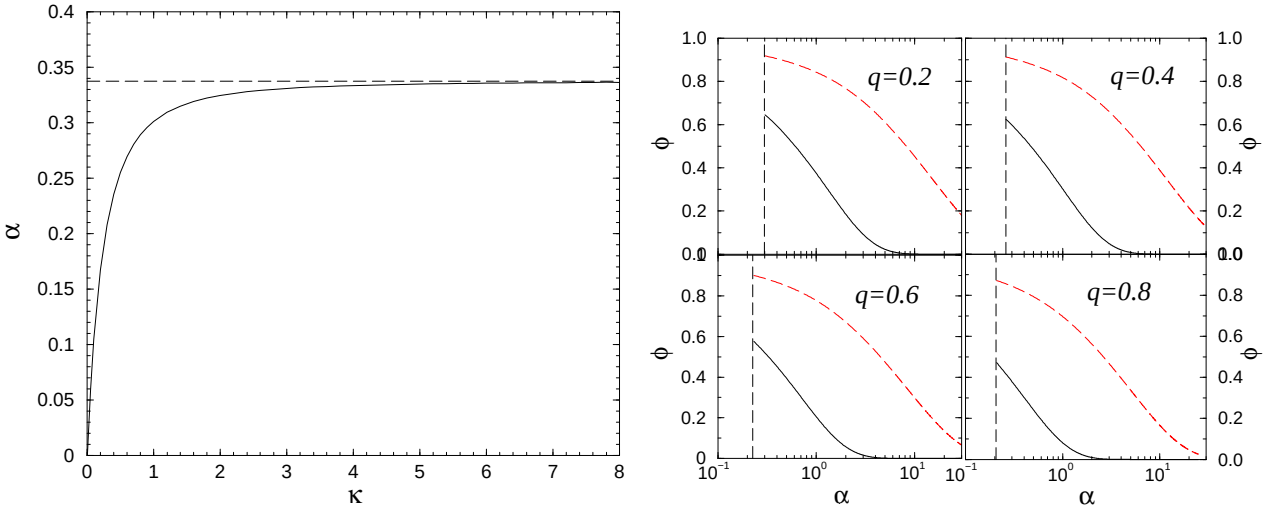
$\alpha = P/N$ (relative number of information patterns)

Regular and occasional traders

Agents with more strategies, trading on different time scales (Marsili & Piai 2003, De Martino 2003)

Power-law distributed trading frequencies

$$P(f) \sim f^{\kappa-1}$$



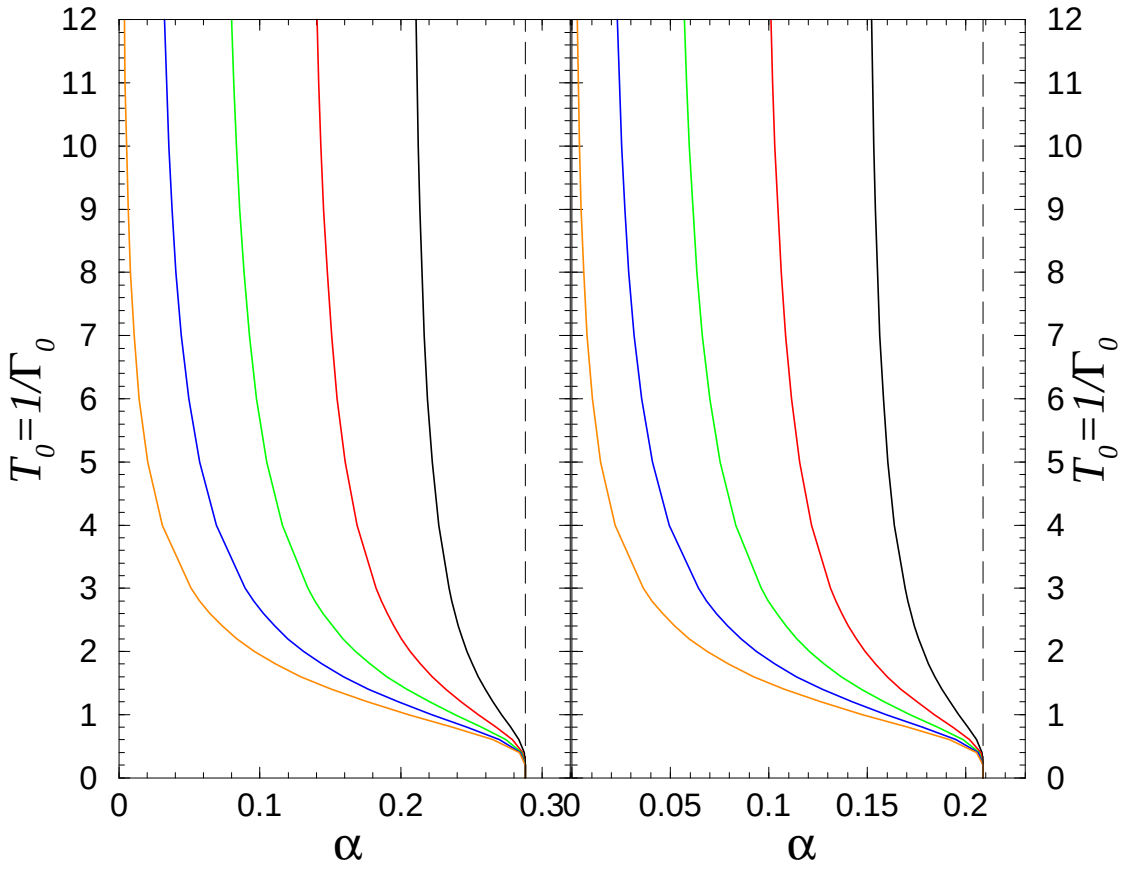
Occasional traders 'freeze' and consitute profitable trading opportunities to regular traders

Agents with different learning rates

From $s_i(t) = \text{sign}[y_i(t)]$ to

$$\text{Prob}\{s_i(t) = \pm 1\} \sim e^{\pm \Gamma \text{sign}[y_i(t)]}$$

Γ = learning rate



Mixed population of deterministic ($\Gamma \to \infty$) and stochastic ($\Gamma = \Gamma_0$) traders.

Left: more regular traders; right: more occasional traders (De Martino 2003)

Fundamentalists and trend-followers

Trend followers in real markets are considered responsible for bubbles, crashes, volatility build-ups, . . .
Ref.: Lux & Marchesi, *Nature* **397** 498 (1999)

$a_i(t)$ action of agent i; $A(t) = \sum_i a_i(t)$

Payoff to i: $a_i(t)\Phi(A(t))$

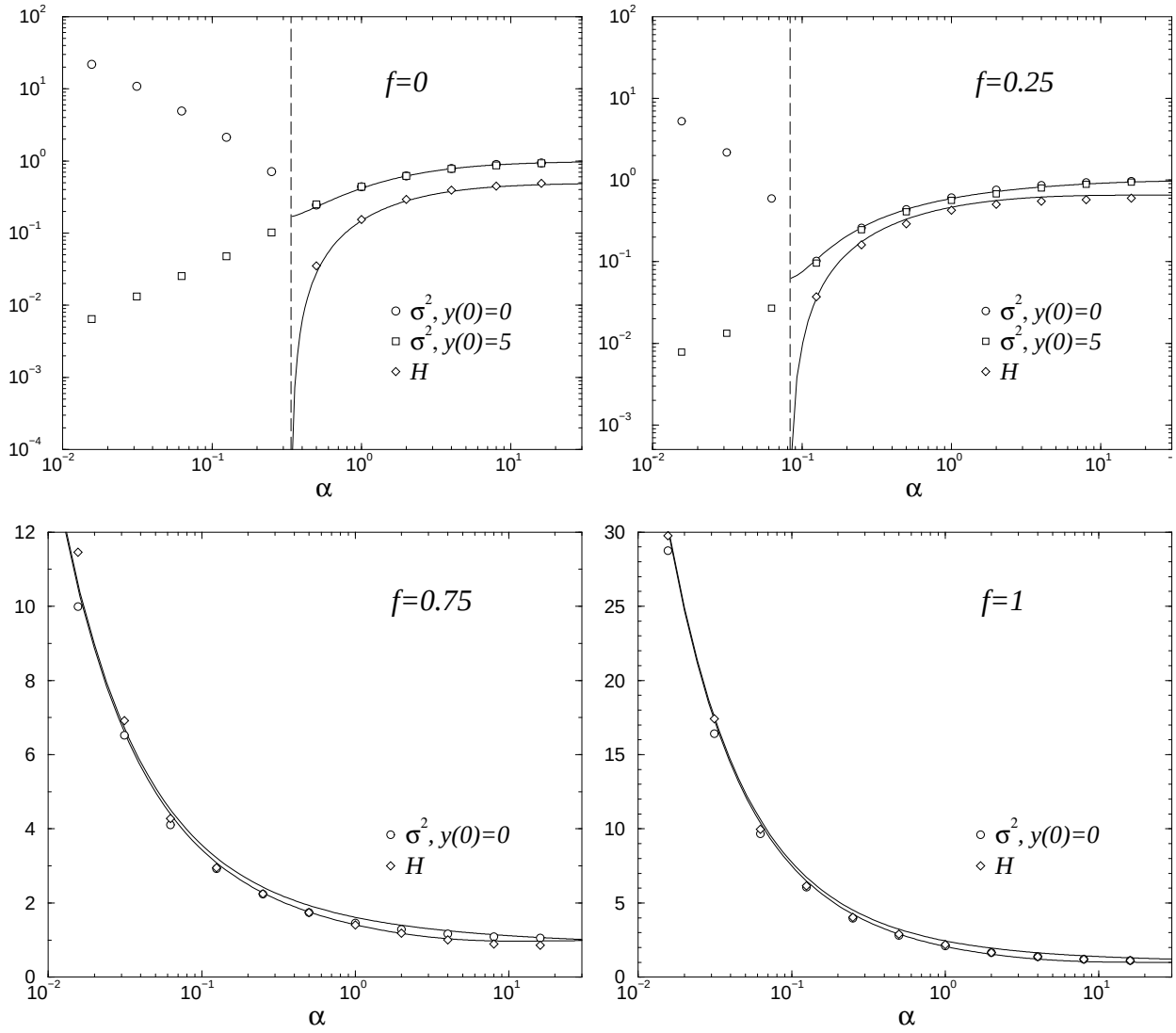
Minority game: $\Phi = -A$ (the minority gains)
$\Rightarrow$ agents are anti-imitative ("Fundamentalists")

Majority game: $\Phi = A$ (the majority gains)
$\Rightarrow$ agents are imitative ("Trend-followers")

Problem: interplay between fundamentalists and trend-followers

Mixed majority-minority game

(De Martino, Giardina & Mosetti 2003)



f = fraction of trend-followers

Epilogue

• Fluctuation phenomena and other macroscopic effects observed in real markets (such as the creation of exploitable information) are not explained by traditional economic theories based e.g. on rational expectations

• Physicists hope that their occurrence can be understood starting from the laws that govern the behavior of the individual agents

• The minority game provides a simple, tractable framework for a large class of market models with the microscopic laws that can be tuned to describe different types of economic agents and their interactions

• These models display a rich behaviour and can reproduce the non-trivial fluctuation phenomena observed in real markets

• I have presented a very biased overview of the field. More (with applications) can be found on

www.unifr.ch/econophysics/minority